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基于改进型离散 Wirtinger 型不等式的 时变时滞系统稳定性分析

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摘 要: 针对离散时变时滞系统的稳定性问题,采用改进型离散 Wirtinger 不等式方法进行研究,获得了一个保守性更低的稳定性判据. 首先构造了一个新的辅助向量函数,利用正定矩阵的二次型都是正值的特性,提出了改进型 Wirtinger 不等式,获得了比传统的方法如离散 Jensen 不等式、离散 Wirtinger 不等式更好的求和逼近结果. 在此基础上,构建了合适的 Lyapunov-Krasovskii 泛函,应用改进型离散 Wirtinger 不等式从而获得有更低保守性的稳定性判据,最终利用两个数值仿真完成验证. 验证结果表明,利用新方法得到的时滞上界要大于现有的不等式方法,更接近理论值,从而表明了本文方法的有效性和优越性.

关键词: 线性离散时滞系统;改进型离散 Wirtinger 不等式;稳定性分析;Lyapunov-Krasovskii 泛函法

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Stability analysis of time-varying delayed systems based on improved discrete wirtinger-based inequality

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Abstract: In view of the stability analysis of linear discrete systems, an intense research has been conducted by adopting an improved discrete Wirtinger-based inequality. A new stability criterion of linear discrete systems with interval time-varying delay is derived, which yields less conservative stability conditions. By definition of a novel auxiliary function, we propose a better integral inequality and receive a tighter stability condition as compared to the recently results such as discrete Jensen inequality, discrete Wirtinger-based inequality. Based on the new inequality, we construct a dedicated Lyapunov-Krasovskii functional and obtain a tighter estimation of the derivative of Lyapunov-Krasovskii functional in terms of the new inequality. Finally, two numerical examples are given to demonstrate the efficiency of our new methods.

Key words: linear discrete time-varying delayed systems; improved discrete Wirtinger-based inequality; stability analysis; Lyapunov-Krasovskii functional

1 引言

实际系统中广泛存在时滞现象,例如常见的工业生产过程、网联系统以及生物集群系统等^[1]. 时滞

的存在容易对系统产生不稳定、震荡等不良影响,影响系统的整体性能^[2]. 因此,针对时滞系统的研究是控制领域的一大重要研究课题.

时滞系统的稳定性分析方法主要分为两类:输

入-输出法^[3]和 Lyapunov-Krasovskii 泛函法^[2-24]. 输入-输出法需要对时滞系统进行模型变换^[3], 方法较为复杂, 因此更为简单有效的 Lyapunov-Krasovskii 泛函法受到了更多的关注. 而 Lyapunov-Krasovskii 泛函法在进行稳定性分析时不可避免会出现保守性问题, 解决的方法主要可以分为两大类: 一类是构造合适的 Lyapunov-Krasovskii 泛函^[4]; 另一类是使用不同的不等式来更好地逼近泛函导数的上界^[5-7]. 实际研究中这两类方法通常混合使用以得到更好的结果.

针对第一类方法, 显然构造的 Lyapunov-Krasovskii 泛函越简单保守性就会相应越大. 为了减少保守性, 现有文献提出的 Lyapunov-Krasovskii 泛函中会包涵额外信息, 如时滞状态量^[5], 高阶积分项^[6], 状态量的高阶微分项^[7]等等. 时滞区间分解法^[15]是另一种可以有效减少保守性的方法.

对于第二类积分不等式方法, 减少保守性的关键在于更加逼近泛函导数的上界. 为了处理 Lyapunov-Krasovskii 泛函导数中的积分项, 有学者提出了自由权矩阵法^[8]和积分不等式法^[5, 10-24]. Jensen 不等式^[9]解决了泛函导数中的一重积分项问题, 其后有人提出 Wirtinger 不等式^[10], 并证明 Jensen 不等式是其特例. 受 Wirtinger 不等式启发, 后来有学者提出了 Wirtinger 不等式的改进形式^[12], 反凸法^[13]和自由矩阵积分不等式^[14]等重要成果. 随后, 上述重要成果被应用于线性离散时滞系统的稳定性分析^[15-24].

Wirtinger 型不等式在线性时滞系统的稳定性分析中取得了不错的效果, 其离散型不等式^[19]随后被发表, 并在线性离散时滞系统的稳定性分析中取得了优秀的结果. 受其启发, 文本提出了改进型离散 Wirtinger 型不等式, 用于时滞系统稳定性研究. 最后的 2 个数值仿真也证明了我们提出方法的优越性, 相对于已有成果具有更低的保守性.

下面给出本文中出现的特殊符号的注释. $\mathbb{R}^n$ 代表实数域的 n 维向量空间; $\mathbb{R}^{n \times m}$ 代表实数域的 $n \times m$ 维向量空间 $\mathbf{P} > 0$ 表示矩阵 $\mathbf{P}$ 是对称正定矩阵; “*” 表示对称矩阵中的对称项; $\mathbf{X}^T$ 表示矩阵的转置. “ $\mathbf{A}^\perp$ ” 表示矩阵 $\mathbf{A}$ 的正交补矩阵.

2 问题描述

考虑线性离散时滞系统如下:

$$\begin{cases} x(k+1) = Ax(k) + A_1x(x-\tau(k)), k \geq 0 \\ x(k) = \phi(k), k = -\tau_{\max}, \dots, 0 \end{cases} \quad (1)$$

式中, $x(k) \in \mathbb{R}^n$ 是状态向量; A, A_1 为已知的系统常数; $\phi(k) \in \mathbb{R}^n, k = -\tau_{\max}, \dots, 0$ 是初始条件. 时变时滞 $\tau(k)$ 是一个满足如下条件的离散函数

$$0 \leq \tau_{\min} \leq \tau(k) \leq \tau_{\max} \quad (2)$$

式中, $\tau_{\min}$ 和 $\tau_{\max}$ 为已知正整数.

以下引理将应用于分析时滞系统(1)的稳定性.

引理 1 对于正整数 a, b 和 k , 有以下等式:

$$\sum_{s=k-b}^{k-a-1} (2s-2k+a+b+1) = 0 \quad (3)$$

$$\sum_{s=k-b}^{k-a-1} (2s-2k+a+b+1)^2 = \frac{(N^2-1)N}{3} \quad (4)$$

引理 2^[12] 对于给定正整数 n, m , 标量 $\alpha \in (0, 1)$, 任意 $n \times m$ 维矩阵 $\mathbf{R} > 0$ 和 $n \times m$ 维矩阵 $\mathbf{W}_1, \mathbf{W}_2$, 对任意向量 $\xi \in \mathbb{R}^m$, 定义函数 $\Theta(\alpha, \mathbf{R})$ 如下

$$\Theta(\alpha, \mathbf{R}) = \frac{1}{\alpha} \xi^T \mathbf{W}_1^T \mathbf{R} \mathbf{W}_1 \xi + \frac{1}{1-\alpha} \xi^T \mathbf{W}_2^T \mathbf{R} \mathbf{W}_2 \xi. \quad (5)$$

如果存在矩阵 $\mathbf{X} \in \mathbb{R}^{n \times n}$ 且 $\begin{bmatrix} \mathbf{R} & \mathbf{X} \\ * & \mathbf{R} \end{bmatrix} > 0$, 则如下

下不等式成立

$$\min_{\alpha \in (0, 1)} \Theta(\alpha, \mathbf{R}) \geq \begin{bmatrix} \mathbf{W}_1 \xi \\ \mathbf{W}_2 \xi \end{bmatrix}^T \begin{bmatrix} \mathbf{R} & \mathbf{X} \\ * & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{W}_1 \xi \\ \mathbf{W}_2 \xi \end{bmatrix}. \quad (6)$$

引理 3^[19] 设 $\zeta \in \mathbb{R}^n, \Phi = \Phi^T \in \mathbb{R}^{n \times n}, \Upsilon \in \mathbb{R}^{m \times n}$ 且 $\text{rank}(\Upsilon) < n$. 则下述条件等价:

- (1) $\zeta^T \Phi \zeta < 0, \forall \Upsilon \zeta = 0, \zeta \neq 0$,
- (2) $\Upsilon^\perp \Phi \Upsilon^\perp < 0$,
- (3) $\exists \mathbf{X} \in \mathbb{R}^{n \times m}: \Phi + \mathbf{X} \Upsilon + \Upsilon^T \mathbf{X}^T < 0$.

引理 4^[18] 对于给定正定矩阵 $\mathbf{R}$ 与三个非负整数 a, b, k 且满足 $a \leq b \leq k$, 设

$$\Gamma(k, a, b) = \begin{cases} \frac{1}{b-a} \left[\left(2 \sum_{s=k-b}^{k-a-1} x(s) \right) + x(k-a) - x(k-b) \right], & a < b \text{ 则有} \\ 2x(k-a), & a = b \end{cases}$$

$$-(b-a) \sum_{s=k-b}^{k-a-1} \Delta x^T(s) \mathbf{R} \Delta x(s) \leq - \begin{bmatrix} \Omega_0 \\ \Omega_1 \end{bmatrix}^T \begin{bmatrix} \mathbf{R} & 0 \\ 0 & 3\mathbf{R} \end{bmatrix} \begin{bmatrix} \Omega_0 \\ \Omega_1 \end{bmatrix} \quad (7)$$

式中:

$$\Omega_0 = x(k-a) - x(k-b)$$

$$\Omega_1 = x(k-a) + x(k-b) - \Gamma(k, a, b)$$

3 主要结果

本文推导出了一种改进型离散 Wirtinger 型不等式,并提出了全新的 Lyapunov-Krasovskii 泛函,最终获得了保守性更低的稳定条件.对定理的说明与证明如下:

3.1 改进型离散 Wirtinger 型不等式

推论 1 对于给定正定矩阵 R 与三个非负整数 a, b, k 且满足 $a \leq b \leq k$, 有下式成立

$$-(b-a) \sum_{s=k-b}^{k-a-1} \Delta x^T(s) R \Delta x(s) \leq - \begin{bmatrix} \omega_0 \\ \omega_1 \end{bmatrix}^T \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \omega_0 \\ \omega_1 \end{bmatrix} \quad (8)$$

式中:

$$\begin{aligned} \omega_0 &= x(k-a) - x(k-b) \\ \omega_1 &= \sqrt{\frac{3N^2}{N^2-1}} \begin{bmatrix} (1-\frac{1}{N})x(k-a) + \\ (1+\frac{1}{N})x(k-b) - \frac{2}{N} \sum_{s=k-b}^{k-a-1} x(s) \end{bmatrix} \\ &= \sqrt{\frac{3N^2}{N^2-1}} [x(k-a) + x(k-b) - \Gamma(k, a, b)] \end{aligned} \quad (9)$$

证明:设

$$\begin{aligned} z(s) &= \\ \Delta x(s) - \frac{1}{N} \omega_0 - \sqrt{\frac{3}{N^2-1}} \frac{2s-2k+a+b+1}{N} \omega_1 \end{aligned} \quad (10)$$

易知

$$z^T(s) R z(s) \geq 0 \quad (11)$$

考虑如下等式

$$\sum_{s=k-b}^{k-a-1} (2s-2k+a+b+1) \Delta x^T(s) = \sqrt{\frac{N^2-1}{3}} \omega_1 \quad (12)$$

$$\sum_{s=k-b}^{k-a-1} \Delta x^T(s) = x^T(k-a) - x^T(k-b) = \omega_0^T \quad (13)$$

对式(11)中的 s 进行求和,则有

$$\begin{aligned} & \sum_{s=k-b}^{k-a-1} z^T(s) R z(s) \\ &= \sum_{s=k-b}^{k-a-1} \Delta x^T(s) R \Delta x(s) - \frac{1}{N} \omega_0^T R \omega_0 - \frac{1}{N} \omega_1^T R \omega_1 \\ &\geq 0 \end{aligned} \quad (14)$$

整理可得

$$-(b-a) \sum_{s=k-b}^{k-a-1} \Delta x^T(s) R \Delta x(s) \leq - \begin{bmatrix} \omega_0 \\ \omega_1 \end{bmatrix}^T \begin{bmatrix} R & \\ & R \end{bmatrix} \begin{bmatrix} \omega_0 \\ \omega_1 \end{bmatrix}$$

证明结束.

注 1 下面证明我们提出的不等式比文献[19]具有更低的保守性.

证明:由式(9)可知

$$\begin{aligned} \omega_1 &= \sqrt{\frac{3N^2}{N^2-1}} \begin{bmatrix} (1-\frac{1}{N})x(k-a) + \\ (1+\frac{1}{N})x(k-b) - \frac{2}{N} \sum_{s=k-b}^{k-a-1} x(s) \end{bmatrix} \\ &= \sqrt{\frac{3N^2}{N^2-1}} [x(k-a) + x(k-b) - \Gamma(k, a, b)] \\ &= \sqrt{\frac{3N^2}{N^2-1}} \Omega_1 \end{aligned}$$

因此

$$\omega_1^T R \omega_1 = \frac{N^2}{N^2-1} \Omega_1^T 3R \Omega_1 > \Omega_1^T 3R \Omega_1 \quad (15)$$

将式(15)代入式(14)中可得

$$-(b-a) \sum_{s=k-b}^{k-a-1} \Delta x^T(s) R \Delta x(s) \leq - \begin{bmatrix} \omega_0 \\ \omega_1 \end{bmatrix}^T \begin{bmatrix} R & \\ & R \end{bmatrix} \begin{bmatrix} \omega_0 \\ \omega_1 \end{bmatrix} < - \begin{bmatrix} \Omega_0 \\ \Omega_1 \end{bmatrix}^T \begin{bmatrix} R & \\ & 3R \end{bmatrix} \begin{bmatrix} \Omega_0 \\ \Omega_1 \end{bmatrix}$$

完成证明.

注 2 显然, $\omega_0^T R \omega_0$ 与 Jensen 不等式的右侧项相同, 并且 $\frac{1}{N} \omega_1^T R \omega_1$ 是非负项, 可知新不等式比 Jensen 不等式具有更低的保守性, 数值仿真结果也证明这一结论.

注 3 很容易验证著名的 Jensen 不等式是我们新不等式的特例. 若在式(10)中设 $\omega_1 = 0$, 则新不等式则退化成 Jensen 不等式.

$$-(b-a) \sum_{s=k-b}^{k-a-1} \Delta x^T(s) R \Delta x(s) \leq - \omega_0^T R \omega_0. \quad (16)$$

3.2 稳定性分析

为简化公式, 定义下列符号:

$$\begin{aligned} \eta(k) &= \\ & \left[x^T(k) \sum_{s=k-\tau_{\min}}^{k-1} x^T(s) \sum_{s=k-\tau_s}^{k-\tau_{\min}-1} x^T(s) \sum_{s=k-\tau_{\max}}^{k-\tau_s-1} x^T(s) \right]^T \\ \xi(k) &= [x^T(k) x^T(k-\tau_k) x^T(k-\tau_{\min}) x^T(k-\tau_s) x^T(k-\tau_{\min}) \Delta x^T(k) \\ & \mu_1^T(k) \mu_2^T(k) \mu_3^T(k) \mu_4^T(k) \mu_5^T(k) \mu_6^T(k) \mu_7^T(k)]^T \end{aligned}$$

$$e_i = [0_{n \times (i-1)n} \quad I_n \quad 0_{n \times (13-i)n}], i = 1, 2, \dots, 13$$

$$\Pi_1 = [e_6^T \quad e_1^T - e_3^T \quad e_3^T - e_4^T \quad e_4^T - e_5^T]^T$$

$$\Pi_2 = \left[e_1^T \quad \frac{1}{2} (e_3^T - e_1^T + \tau_{\min} e_7^T) \right. \\ \left. \Pi_3^T \frac{1}{2} (e_5^T - e_4^T + (\tau_{\max} - \tau_s) e_9^T) \right]^T$$

$$\Pi_3 = \frac{1}{2} [(\tau_s - \tau(k)) e_{10}^T + (\tau(k) - \tau_{\min}) e_{11}^T - e_3^T + e_4^T]^T$$

$$\Pi_4 = \left[e_1^T - e_3^T \quad \sqrt{\frac{3\tau_{\min}^2}{\tau_{\min}^2 - 1}} (e_1^T + e_3^T - e_7^T) \right]^T$$

$$\Pi_6 = \left[e_3^T - e_2^T \sqrt{\frac{3(\tau(k) - \tau_{\min})^2}{(\tau(k) - \tau_{\min})^2 - 1}} (e_3^T + e_2^T - e_{11}^T) \right]^T$$

$$\Pi_7 = [\Pi_5^T \quad \Pi_6^T]^T$$

$$\Pi_8 = \left[e_4^T - e_5^T \sqrt{\frac{3(\tau_{\max} - \tau_s)^2}{(\tau_{\max} - \tau_s)^2 - 1}} (e_4^T + e_5^T - e_9^T) \right]^T$$

$$\Pi_9 = \frac{1}{2} \left[(\tau_{\max} - \tau(k)) e_{12} + (\tau(k) - \tau_s) e_{13} - e_4 + e_5 \right]$$

$$\Pi_{10} = \left[e_1^T \frac{1}{2} (e_3^T - e_1^T + \tau_{\min} e_7^T) \right. \\ \left. \frac{1}{2} (e_4^T - e_3^T + (\tau_s - \tau_{\min}) e_8^T) \Pi_9^T \right]^T$$

$$\Pi_{11} = \left[e_3^T - e_4^T \sqrt{\frac{3(\tau_s - \tau_{\min})^2}{(\tau_s - \tau_{\min})^2 - 1}} (e_3^T + e_4^T - e_8^T) \right]^T$$

$$\Pi_{12} =$$

$$\left[e_4^T - e_2^T \sqrt{\frac{3(\tau(k) - \tau_s)^2}{(\tau(k) - \tau_s)^2 - 1}} (e_4^T + e_2^T - e_{13}^T) \right]^T$$

$$\Pi_{13} = \left[e_2^T - e_5^T \sqrt{\frac{3(\tau_{\max} - \tau(k))^2}{(\tau_{\max} - \tau(k))^2 - 1}} (e_2^T + e_5^T - e_{12}^T) \right]^T$$

$$\Pi_{14} = [\Pi_{12}^T \quad \Pi_{13}^T]^T$$

$$\Upsilon = [A - I_n \quad A_1 \quad 0_{n \times 3n} \quad -I_n \quad 0_{n \times 7n}]$$

$$\mu_1(k) = \Gamma(k, 0, \tau_{\min})$$

$$\mu_2(k) = \Gamma(k, \tau_{\min}, \tau_s)$$

$$\mu_3(k) = \Gamma(k, \tau_s, \tau_{\max})$$

$$\mu_4(k) = \Gamma(k, \tau(k), \tau_s)$$

$$\mu_5(k) = \Gamma(k, \tau_{\min}, \tau(k))$$

$$\mu_6(k) = \Gamma(k, \tau(k), \tau_{\max})$$

$$\mu_7(k) = \Gamma(k, \tau_s, \tau(k))$$

定理 1 系统(1) 渐进稳定的充分条件是存在 $4n \times 4n$ 维的正定矩阵 P , 6 个 $n \times n$ 维正定矩阵 $Q_1, Q_2, Q_3, R_1, R_2, R_3$ 和两个 $2n \times 2n$ 维的矩阵 Y_2, Y_3 , 则下列 LMI 成立:

$$\Upsilon^\perp \Xi_1(\tau(k)) \Upsilon^\perp \leq 0, \forall \tau(k) \in \{\tau_{\min}, \tau_s\} \quad (17)$$

$$\Upsilon^\perp \Xi_2(\tau(k)) \Upsilon^\perp \leq 0, \forall \tau(k) \in \{\tau_s, \tau_{\max}\} \quad (18)$$

$$\Theta_i = \begin{bmatrix} \bar{R}_i & Y_i \\ * & \bar{R}_i \end{bmatrix} > 0, i = 2, 3 \quad (19)$$

式中:

$$\bar{R}_i = \begin{bmatrix} R_i & 0 \\ 0 & R_i \end{bmatrix}, i = 1, 2, 3$$

$$\Xi_1(\tau(k)) = \Pi_1^T P \Pi_1 + 2\Pi_1^T P \Pi_2 + e_1^T Q_1 e_1 - e_5^T Q_3 e_5 + e_3^T (Q_2 - Q_1) e_3 + e_4^T (Q_3 - Q_2) e_4 - \Pi_4^T \bar{R}_1 \Pi_4$$

$$- \Pi_7^T \Theta_2 \Pi_7 - \Pi_8^T \bar{R}_3 \Pi_8 + e_6^T (\tau_{\min}^2 R_1 + (\tau_s - \tau_{\min})^2 R_2 + (\tau_{\max} - \tau_s)^2 R_3) e_6$$

$$\Xi_2(\tau(k)) = \Pi_1^T P \Pi_1 + 2\Pi_1^T P \Pi_2 + e_1^T Q_1 e_1 - e_5^T Q_3 e_5 + e_3^T (Q_2 - Q_1) e_3 + e_4^T (Q_3 - Q_2) e_4 - \Pi_4^T \bar{R}_1 \Pi_4 -$$

$$\Pi_{11}^T \bar{R}_2 \Pi_{11} - \Pi_{14}^T \Theta_3 \Pi_{14} + e_6^T (\tau_{\min}^2 R_1 + (\tau_s - \tau_{\min})^2 R_2 + (\tau_{\max} - \tau_s)^2 R_3) e_6$$

证明:构造如下 Lyapunov - Krasovskii 泛函:

$$V = V_1 + V_2 + V_3 \quad (20)$$

式中:

$$V_1 = \eta(k)^T P \eta(k),$$

$$V_2 = \sum_{s=k-\tau_{\min}}^{k-1} x^T(s) Q_1 x(s) \\ + \sum_{s=k-\tau_s}^{k-\tau_{\min}-1} x^T(s) Q_2 x(s) + \sum_{s=k-\tau_{\max}}^{k-\tau_s-1} x^T(s) Q_3 x(s),$$

$$V_3 = \tau_{\min} \sum_{s=-\tau_{\min}}^{-1} \sum_{v=k+s}^{k-1} \Delta x^T(v) R_1 \Delta x(v) \\ + (\tau_s - \tau_{\min}) \sum_{s=\tau_s}^{\tau_{\min}-1} \sum_{v=k+s}^{k-1} \Delta x^T(v) R_2 \Delta x(v) \\ + (\tau_{\max} - \tau_s) \sum_{s=-\tau_{\max}}^{\tau_s-1} \sum_{v=k+s}^{k-1} \Delta x^T(v) R_3 \Delta x(v)$$

由 $\|\eta(k)\| \geq \|x(k)\|$ 可得

$$V \geq \lambda_{\min}(P) \|x(k)\|^2 \quad (21)$$

对 V 求前向差分, 可得

$$\Delta V_1 = \Delta \eta^T(k) P \Delta \eta(k) + 2\Delta \eta^T(k) P \eta(k) \\ = \xi^T(k) \{ \Pi_1^T P \Pi_1 + 2\Pi_1^T P \Pi_2 \} \xi(k) \quad (22)$$

$$\Delta V_2 = \xi^T(k) \left\{ e_1^T Q_1 e_1 + e_3^T (Q_2 - Q_1) e_3 \right. \\ \left. + e_4^T (Q_3 - Q_2) e_4 - e_5^T Q_3 e_5 \right\} \xi(k) \quad (23)$$

$$\Delta V_3 = \Delta x^T(k) \left[\tau_{\min}^2 R_1 + (\tau_s - \tau_{\min})^2 R_2 \right. \\ \left. + (\tau_{\max} - \tau_s)^2 R_3 \right] x(k)$$

$$- \tau_{\min} \sum_{s=k-\tau_{\min}}^{k-1} \Delta x^T(s) R_1 \Delta x(s) \\ - (\tau_s - \tau_{\min}) \sum_{s=k-\tau_s}^{k-\tau_{\min}-1} \Delta x^T(s) R_2 \Delta x(s) \\ - (\tau_{\max} - \tau_s) \sum_{s=k-\tau_{\max}}^{k-\tau_s-1} \Delta x^T(s) R_3 \Delta x(s), \quad (24)$$

针对时滞系统(1) 分两种情况进行讨论:(I) $\tau_{\min} \leq \tau(k) \leq \tau_s$ 和(II) $\tau_s \leq \tau(k) \leq \tau_{\max}$. 对于(I),由式(8),推论 1 和引理 2,可得

$$-\tau_{\min} \sum_{s=k-\tau_{\min}}^{k-1} \Delta x^T(s) R_1 \Delta x(s) \leq -\xi^T(k) \Pi_4^T \bar{R}_1 \Pi_4 \xi(k)$$

(25)

$$-(\tau_s - \tau_{\min}) \sum_{s=k-\tau_s}^{k-\tau_{\min}-1} \Delta x^T(s) R_2 \Delta x(s) \leq -\xi^T(k) \Pi_7^T \Theta_2 \Pi_7 \xi(k)$$

(26)

$$-(\tau_{\max} - \tau_s) \sum_{s=k-\tau_{\max}}^{k-\tau_s-1} \Delta x^T(s) R_3 \Delta x(s) \leq -\xi^T(k) \Pi_8^T \bar{R}_3 \Pi_8 \xi(k)$$

(27)

由式(22-27),可得

$$\Delta V \leq \xi^T(k) \Xi_1 \xi(k).$$

(28)

尽管 $\Upsilon^{\perp T} \Xi_1(\tau(k)) \Upsilon^{\perp}$ 是 $\tau(k)$ 的非线性函数,鉴于 $\tau(k)$ 是在 $[\tau_{\min}, \tau_s]$ 区间上的离散函数,可以用遍历法求得函数值.

利用引理 3,可得

$$\Delta V \leq \Upsilon^{\perp T} \Xi_1(\tau(k)) \Upsilon^{\perp} \leq 0,$$

$$\forall \tau(k) \in \{\tau_m, \dots, \tau_a\}$$

(29)

对于(II),类似可得

$$\Delta V \leq \xi^T(k) \Xi_2 \xi(k) \leq 0,$$

$$\forall \tau(k) \in \{\tau_s, \dots, \tau_{\max}\}.$$

(30)

结合式(21)(29)(30),可以证明时滞系统(1) 渐进稳定. 完成证明.

注 4 定理 1 的变量数为 $19n^2 + 5n$.

4 数值仿真

本节给出了两个经典系统的数值仿真结果并与现有方法进行了对比说明了其优越性.

例 1 考虑如下时滞系统(1),其中

$$A = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, A_1 = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}.$$

随 $\tau_{\min}$ 变化的使系统稳定的最大时滞上界值 $\tau_{\max}$ 如表 1 所示. 对比现有成果,尤其是文献[19] 的结果,本文得到的稳定性判据定理 1 拥有最大的时滞上界,仿真结果再次证明了改进型离散 Wirtinger 不等式在减少保守性上的优势.

例 2 改变时滞系统(1) 的参数为

$$A = \begin{bmatrix} 0.7 & 0.1 \\ 0.05 & 0.7 \end{bmatrix}, A_1 = \begin{bmatrix} -0.1 & 0.1 \\ -0.1 & -0.2 \end{bmatrix}.$$

类似的,表 2 列出了随 $\tau_{\min}$ 变化的使系统稳定的

最大时滞上界值 $\tau_{\max}$. 从仿真结果可知,本文中提出的稳定性条件得到的时滞上界最大,整体趋势与表 1 一致,可以作为表 1 的补充说明,再次证明了新不等式在减少保守性上的优越性.

表 1 随 $\tau_{\min}$ 变化的最大时滞上界值 $\tau_{\max}$ (例 1)

方法 \ $\tau_{\min}$	2	6	10	15	25	30	变量个数
[17]	13	14	15	18	26	30	143
[18]	13	14	17	20	29	33	42
[22]	17	18	20	23	31	35	22
[20]	17	18	20	23	31	35	42
[24](定理 1)	19	20	21	24	31	35	31
[15](定理 3)	20	20	21	24	31	35	76
[21](定理 1)	17	18	20	24	31	36	117
[19](定理 1)	22	22	23	26	32	36	86
定理 1	23	25	26	28	36	40	86

表 2 随 $\tau_{\min}$ 变化的最大时滞上界值 $\tau_{\max}$ (例 2)

方法 \ $\tau_{\min}$	2	5	6	7	10	20	变量个数
[18]	7	9	10	11	14	24	42
[22]	9	11	12	13	16	26	22
[23]	9	11	12	13	16	26	42
[24](定理 1)	12	14	15	16	19	29	76
[15](定理 3)	12	14	15	16	19	29	76
[21](定理 1)	9	12	13	14	17	27	117
[19](定理 1)	14	17	18	18	21	31	86
定理 1	16	18	19	20	23	33	86

5 结束语

基于离散 Wirtinger 不等式,本文提出了其改进型不等式,且应用于线性离散时滞系统的稳定性分析当中,最终得到一个新的稳定性判据. 本文不仅推导证明了改进型离散 Wirtinger 型不等式具有更低的保守性,还通过两个数值仿真证明了新不等式可以降低系统的保守性,使时滞积分上界更趋近于理论值,仿真结果对比现有的不等式方法,证明了该方法的优越性.

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