

随机发生非线性与测量时滞的滤波算法设计

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摘 要: 针对一类具有随机发生非线性和一步测量时滞的时变系统的滤波问题。通过引入服从伯努利分布的随机序列, 描述随机发生非线性与一步测量时滞。与此同时, 引入事件触发传输机制且对所提出系统进行增广构造出滤波器。从而提出一种具有一步测量时滞与随机发生非线性的滤波算法。使同时存在一步测量时滞、噪声和随机发生非线性的情况下, 可以采用放缩找到滤波误差协方差矩阵的上界, 并且通过设计滤波增益矩阵使得该上界的迹达到最小。最后, 利用 matlab 算例仿真, 验证所提出滤波算法的真实性与实用性。

关键词: 时变离散系统; 一步测量时滞; 随机发生非线性; 滤波

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Design on Filtering Algorithm with Random Nonlinearity and One-step Measurement Delay

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Abstract: This paper studies the filtering problem of a class of time-varying systems with random nonlinearity and one-step measurement delay. The random nonlinearity and one-step measurement delay are described by introducing the random sequences obeying Bernoulli distribution. At the same time, the event-triggered transmission mechanism is introduced and the proposed system is augmented to construct a filter. In this paper, a filtering algorithm with one-step measurement delay and random nonlinearity is proposed. When one-step measurement delay, noise and random nonlinearity exist at the same time, the upper bound of the covariance matrix of filtering error is found by scaling, and the trace of the upper bound is minimized by designing the filter gain matrix. Finally, the validity and practicability of the proposed filtering algorithm are verified by matlab simulation.

Keywords: discrete time-varying systems; one-step measurement delay; random nonlinearity; filter

0 引 言

作为现代控制理论的一个重要分支, 卡尔曼滤

波^[1]得到了国内外专家学者的广泛研究。卡尔曼滤波是一种算法, 且该算法具有递推形式。卡尔曼滤波算法的优点在于, 其本身是一种递推估计算法, 不需要存储所有的观测信息, 只需上一个估计时刻

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以及当前时刻的信息即可求出当前时刻的估计值,而且计算相对方便,存储量也相对较小。但在实际工程中,传统的卡尔曼滤波的缺点又是显而易见的。如受外界因素的影响,我们并不能充分了解噪声的统计特性,从而无法实现对卡尔曼滤波的最优估计;处于实际运动环境中,所建立模型与实际系统一般具有差异性。事实上,与日渐完善的线性系统相比,在实际环境中,系统往往是非线性的,并且有很多都是随机发生的。文 [2] 研究了一类带有随机非线性与测量丢失的滤波问题。此外,考虑到随机发生概率的不确定性,文 [3] 探讨了不确定概率下带有随机非线性状态估计问题。带有随机发生非线性系统的卡尔曼滤波问题也引起了广泛的关注 [4-8]。

在实际问题中,由于信息在传输过程中存在环境或技术因素的影响,必然会引发传感器的时滞现象,影响网络化控制系统的性能,在非线性和系统中表现的更为突出 [9]。文 [10] 叙述了时滞现象的不确定性,通过已知概率信息,处理了具有时变系统的渐进均方稳定问题。对于如何解决系统中的时滞现象已成为近年来学者们研究的热点 [11-13]。其中,文 [5] 为填补网络延迟对系统的影响,采用了模糊逻辑调剂方法。文 [6] 为求解随机有界时滞的状态估计问题尝试性地通过增广的方式设计出了最小方差估计器。此外,文 [7] 考虑同时具有时滞与测量数据丢失的网络控制系统,探究了非脆弱 L_1 滤波问题。

当数据通过媒介进行交换和传输时,由于网络带宽资源受限,往往会发生各类的网络诱导现象。为了减少网络传输压力,节省网络带宽资源,在实际网络化传输系统中,往往会加入相应的传输协议,如 Round-Robin 协议 [18-20], Try-Once-Discard 协议 [21-23] 等,本文中所考虑的是事件触发传输协议。在以往的研究中,研究者们考虑的往往都是时间触发协议,但是时间触发协议不能充分利用有限的网络资源,而且在进行数据传输时,数据中包含的新信息特别有限,对原本就有限的网络资源造成了极大浪费。不同于时间触发,事件触发的基本思想是“按需分配”,只有在系统满足一定的条件时,数据才会进行传递。本文引入事件触发协议,可以有效地利用通信资源和维护系统稳定。其中为针对时变的网络诱导时滞,保证系统的有界输入-有界输出稳定性,文 [24] 提出了基于事件触发机制的控制策略。文 [25] 又通过基于事件触发机制的自适应差分调制方法,比较有效地解决了在数据丢失情形下

的控制问题。

鉴于上述讨论,本文目的是提出一种鲁棒滤波方法,用于传感器网络上具有时滞、测量噪声和随机发生非线性的离散时变系统。利用伯努利分布随机变量,描述一步测量时滞与随机发生非线性现象。本文主要贡献如下:①研究测量时滞、测量噪声以及随机发生非线性同时存在的情况下离散时变系统,所考虑的模型更具一般性;②给出了具有随机发生非线性的系统的滤波方法,通过求解两个类黎卡提差分方程,得到了滤波误差协方差上界,并且设计了适当的滤波器参数使上界的迹达到最小。

2 模型建立

考虑具有随机发生非线性和传感器随机一步测量时滞的离散时变动态系统:

$$\left. \begin{aligned} \vec{x}_{k+1} &= \bar{A}_k \vec{x}_k + \alpha_k \bar{f}(\vec{x}_k) + \bar{B}_k \vec{\omega}_k \\ \vec{y}_k &= \bar{C}_k \vec{x}_k + \vec{v}_k \\ y_k &= \lambda_k \vec{y}_k + (1 - \lambda_k) \vec{y}_{k-1} \end{aligned} \right\} \quad (1)$$

其中: $\vec{x}_k$ 代表 k 时刻系统的状态向量; $\vec{y}_k$ 代表系统的测量输出; $\bar{f}(\vec{x}_k)$ 为非线性函数; $\vec{\omega}_k$ 是均值为零方差为 $\bar{Q}_k$ 的过程噪声; $\vec{v}_k$ 是均值为零方差为 $\bar{R}_k$ 的测量噪声; $\bar{A}_k$ 、 $\bar{B}_k$ 、 $\bar{C}_k$ 均为已知的系统矩阵; α_k 和 λ_k 均为服从伯努利分布的随机变量,分别刻画随机发生的非线性与随机发生的一步测量时滞,并假设其满足以下条件:

$$\left. \begin{aligned} \text{Prob}\{\alpha_k = 1\} &= \{\alpha_k\} = \bar{\alpha}_k & \text{Prob}\{\alpha_k = 0\} &= 1 - \bar{\alpha}_k \\ \text{Prob}\{\lambda_k = 1\} &= \{\lambda_k\} = \bar{\lambda}_k & \text{Prob}\{\lambda_k = 0\} &= 1 - \bar{\lambda}_k \end{aligned} \right\} \quad (2)$$

其中 $\bar{\alpha}_k$ 和 $\bar{\lambda}_k$ 分别代表已知的发生概率。

假设: 非线性函数 $\bar{f}(\vec{x}_k)$ 满足如下的利普希茨条件:

$$\|\bar{f}(\vec{x}_k) - \bar{f}(\vec{z}_k)\| \leq l \|\vec{x}_k - \vec{z}_k\| \quad (3)$$

为了计算简便,进入如下形式的增广:

$$\begin{aligned} x_k &= \begin{bmatrix} \vec{x}_k \\ \vec{x}_{k-1} \end{bmatrix}, f(x_k) = \begin{bmatrix} \bar{f}(\vec{x}_k) \\ 0 \end{bmatrix}, \bar{I} = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}, \\ A_k &= \begin{bmatrix} \bar{A}_k & 0 \\ 0 & 0 \end{bmatrix}, B_k = \begin{bmatrix} \bar{B}_k \\ 0 \end{bmatrix}, C_k = \begin{bmatrix} \bar{C}_k & 0 \\ 0 & \bar{C}_{k-1} \end{bmatrix}, \\ v_k &= \begin{bmatrix} \vec{v}_k \\ \vec{v}_{k-1} \end{bmatrix}, \end{aligned}$$

$$Y_k = [\lambda_k I \quad (1 - \lambda_k) I], \vec{\omega}_k = \omega_k$$

得到增广后的离散时变系统模型:

$$\begin{aligned} x_{k+1} &= \bar{A}_k x_k + \alpha_k f(x_k) + B_k \omega_k \\ \bar{y}_k &= Y_k (C_k x_k + \nu_k) \end{aligned} \quad (4)$$

其中 $\bar{A}_k = (A_k + I)$, 增广后的测量噪声 ν_k 具有如下的统计特性:

$$\begin{aligned} E\{\nu_k\} &= 0 \\ E\{\nu_k \nu_l^T\} &= R_k \delta_{k-l} + R_{k,k+1} \delta_{k-l-1} + R_{k,k+1} \delta_{k-l+1} \end{aligned}$$

其中,

$$\begin{aligned} R_k &= \begin{bmatrix} \bar{R}_k & 0 \\ 0 & \bar{R}_{k-1} \end{bmatrix}, R_{k,k-1} = \begin{bmatrix} 0 & 0 \\ \bar{R}_{k-1} & 0 \end{bmatrix}, R_{k,k+1} \\ &= \begin{bmatrix} 0 & \bar{R}_k \\ 0 & 0 \end{bmatrix} \end{aligned}$$

令 Q_k 为 ω_k 的协方差, 也就是说 $Q_k = \bar{Q}_k$ 。

在网络传输过程中, 为了减少网络传输压力, 节省网络带宽资源, 通常会引入相应的通信协议。在本文中, 引入如下形式的事件触发传输机制:

$$(y_{k+l} - y_{k_i})^T (y_{k+l} - y_{k_i}) > \theta \quad (5)$$

式中 y_{k_i} 是最近事件触发时刻的测量输出, θ 是已知的调节阈值, 那么离散时变系统在 k 时刻的实际输出如下所示:

$$\tilde{y}_k = y_{k_i}, k \in \{k_i, k_i + 1, \dots, k_{i+1} - 1\}$$

其中 $\tilde{y}_k$ 为 k 时刻的实际输出值。

针对上述增广系统, 构造如下形式的滤波器:

$$\begin{aligned} \hat{x}_{k+1|k} &= \bar{A}_k \hat{x}_{k|k} + \bar{\alpha}_k f(\hat{x}_{k|k}) \\ \hat{x}_{k+1|k+1} &= \hat{x}_{k+1|k} + K_{k+1} (\tilde{y}_{k+1} - \bar{Y}_{k+1} C_{k+1} \hat{x}_{k+1|k}) \end{aligned} \quad (6)$$

式中: $\hat{x}_{k|k}$ 是 x_k 在 k 时刻的状态估计; $\hat{x}_{k+1|k}$ 是 x_k 在 k 时刻的一步预测; $\hat{x}_{k+1|k+1}$ 是 $k+1$ 时刻的状态估计; K_{k+1} 是 $k+1$ 时刻的滤波增益矩阵, $[\bar{\lambda}_{k+1} I (1 - \bar{\lambda}_{k+1})] \bar{I} = \bar{Y}_{k+1}$ 。

本文主要有以下两个目的。第一, 针对具有随机发生非线性的离散时变系统(1)设计形如式(6)的滤波器, 得到滤波误差协方差矩阵的上界, 即找到正定矩阵 $\Sigma_{k+1|k+1}$ 满足如下关系式:

$$\{ (x_{k+1} - \hat{x}_{k+1|k+1}) (x_{k+1} - \hat{x}_{k+1|k+1})^T \} \leq \Sigma_{k+1|k+1} \quad (7)$$

第二, 通过设计适当的滤波器增益矩阵 K_{k+1} 使得滤波误差协方差矩阵上界的迹 $\text{tr}(\Sigma_{k+1|k+1})$ 达到最小。

2 主要结论

首先, 介绍如下引理:

引理1^[6] 对于适当维数的的矩阵 M, N, X 和 H , 有如下结果:

$$\begin{aligned} \frac{\partial \text{tr}\{XM\}}{\partial X} &= M^T, \frac{\partial \text{tr}\{MX^T\}}{\partial X} = M, \frac{\partial \text{tr}\{MXN\}}{\partial X} \\ &= M^T N^T \\ \frac{\partial \text{tr}\{MX^T N\}}{\partial X} &= NM, \frac{\partial \text{tr}\{XMX^T\}}{\partial X} = 2XM \\ \frac{\partial \text{tr}\{MXNX^T H\}}{\partial X} &= M^T N^T X N^T + HMXN \\ \frac{\partial \text{tr}\{(MXN)P(MXN)^T\}}{\partial X} &= 2M^T MXNPN^T \end{aligned}$$

其中 P 是任意的对称矩阵。

引理2^[7] 对于两个实列向量 $a, b \in R^n$, 则下面的不等式成立:

$$ab^T + ba^T \leq \varepsilon aa^T + \varepsilon^{-1} bb^T$$

其中 ε 是已知的正数。

根据式(4)、(6), 可得一步预测误差表达式如下:

$$\begin{aligned} \tilde{x}_{k+1|k} &= \bar{A} \tilde{x}_{k|k} + \bar{\alpha}_k [f(x_k) - f(\hat{x}_{k|k})] + \bar{\alpha}_k f(\hat{x}_{k|k}) + \\ &B_k \omega_k \end{aligned} \quad (8)$$

式中 $\bar{\alpha}_k = \alpha_k - \alpha_k$ 。

同样地, 得到滤波误差表达式:

$$\begin{aligned} \tilde{x}_{k+1|k+1} &= (I - K_{k+1} \bar{Y}_{k+1} C_{k+1}) \tilde{x}_{k+1|k} + K_{k+1} (\tilde{y}_{k+1} - \\ &y_{k+1}) + K_{k+1} \tilde{Y}_{k+1} C_{k+1} x_{k+1} - K_{k+1} Y_{k+1} \nu_{k+1} \end{aligned} \quad (9)$$

其中 $\tilde{Y}_{k+1} = Y_{k+1} - \bar{Y}_{k+1}$ 。

引理3 增广系统(4)的状态协方差矩阵 $X_{k+1} = \{x_{k+1} x_{k+1}^T\}$ 具有如下上界:

$$\begin{aligned} X_{k+1} &\leq (1 + \varepsilon) \bar{A}_k X_k \bar{A}_k^T + (1 + \varepsilon^{-1}) l^2 \text{tr}(\bar{X}_k) I \\ &+ B_k Q_k B_k^T \\ &:= \bar{X}_{k+1} \end{aligned} \quad (10)$$

上式中 ε 是已知的正数。

证明: 增广系统(4)的状态协方差矩阵 $X_{k+1} = \{x_{k+1} x_{k+1}^T\}$ 可计算如下:

$$\begin{aligned} X_{k+1} &= \bar{A}_k X_k \bar{A}_k^T + \bar{\alpha}_k \{f(x_k) f^T(x_k)\} + B_k Q_k B_k^T + \\ &\Lambda_1 + \Lambda_1^T \end{aligned} \quad (11)$$

其中 $\Lambda_1 = \{\alpha_k f(x_k) x_k^T \bar{A}_k^T\}$

应用引理2 可得

$$\begin{aligned} \Lambda_1 + \Lambda_1^T &\leq \varepsilon \bar{A}_k X_k \bar{A}_k^T + \varepsilon^{-1} \bar{\alpha}_k \{f(x_k) f^T(x_k)\} \end{aligned} \quad (12)$$

将上式代入到式(11)中,得

$$X_{k+1} \leq (1 + \varepsilon) \bar{A}_k X_k \bar{A}_k^T + (1 + \varepsilon^{-1}) \bar{\alpha}_k \{f(x_k) - f(\hat{x}_{k|k})\} f^T(x_k) + B_k Q_k B_k^T \quad (13)$$

根据不等式性质得到

$$f(x_k) f^T(x_k) \leq \|f(x_k)\|^2 I = f^T(x_k) f(x_k) I \quad (14)$$

$$\{f(x_k) f^T(x_k)\} \leq l^2 \{x_k x_k^T\} = l^2 \text{tr}(X_k) I \quad (15)$$

将上式代入式(13),得到增广系统的状态协方差上界表达式

$$X_{k+1} \leq (1 + \varepsilon) \bar{A}_k X_k \bar{A}_k^T + (1 + \varepsilon^{-1}) l^2 \text{tr}(X_k) I + B_k Q_k B_k^T \quad (16)$$

定理1 一步预测误差协方差矩阵 $P_{k+1|k} =$

$\{\bar{x}_{k+1|k} \bar{x}_{k+1|k}^T\}$ 的递推表达式如下

$$P_{k+1|k} = \bar{A}_k P_{k|k} \bar{A}_k^T + \bar{\alpha}_k^2 \{[f(x_k) - f(\hat{x}_{k|k})] \times [f(x_k) - f(\hat{x}_{k|k})]^T\} + \bar{\alpha}_k(1 - \bar{\alpha}_k) \{f(x_k) f^T(x_k)\} + B_k Q_k B_k^T + A_2 + A_2^T \quad (17)$$

其中, $A_2 = \{\bar{\alpha}_k [f(x_k) - f(\hat{x}_{k|k})] \bar{x}_{k|k}^T \bar{A}_k^T\}$, $P_{k|k} = \{\bar{x}_{k|k} \bar{x}_{k|k}^T\}$ 为滤波误差协方差。

证明: 根据一步预测误差表达式(8),利用 $\{\bar{\alpha}_k\} = 0$, $\{\omega_k\} = 0$,很容易推得式(8),从略。

定理2: 滤波误差协方差 $P_{k+1|k+1} = \{\bar{x}_{k+1|k+1} \times \bar{x}_{k+1|k+1}^T\}$ 的递推表达式如下:

$$P_{k+1|k+1} = (I - K_{k+1} \bar{Y}_{k+1} C_{k+1}) P_{k+1|k} (I - K_{k+1} \bar{Y}_{k+1} C_{k+1})^T + K_{k+1} \{(\bar{y}_{k+1} - y_{k+1})(\bar{y}_{k+1} - y_{k+1})^T\} K_{k+1}^T + \{(K_{k+1} \bar{Y}_{k+1} C_{k+1} x_{k+1})(K_{k+1} \bar{Y}_{k+1} C_{k+1} x_{k+1})^T\} + K_{k+1} \{Y_{k+1} \nu_{k+1} \nu_{k+1}^T Y_{k+1}\} K_{k+1}^T + A_3 + A_4 + A_3^T + A_4^T \quad (18)$$

其中 $A_3 = \{(I - K_{k+1} \bar{Y}_{k+1} C_{k+1}) \bar{x}_{k+1|k} (\bar{y}_{k+1} - y_{k+1})^T K_{k+1}^T\}$,

$$A_4 = \{K_{k+1} (\bar{y}_{k+1} - y_{k+1}) \nu_{k+1}^T Y_{k+1}^T K_{k+1}^T\}$$

证明: 考虑到 $\{\bar{Y}_k\} = 0$ 和 $\{\nu_k\} = 0$,并且根据滤波误差表达式(9),定理2容易得出,故而证明从略。

定理3 对于正数 η, μ_1, μ_2 , 如果如下的类黎卡提差分方程:

$$\begin{aligned} \Sigma_{k+1|k} &= (1 + \eta) \bar{A}_k \Sigma_{k|k} \bar{A}_k^T + (1 + \eta^{-1}) \bar{\alpha}_k^2 l^2 \text{tr}(\Sigma_{k|k}) I + \\ &\bar{\alpha}_k(1 - \bar{\alpha}_k) l^2 \text{tr}(\bar{X}_k) I + B_k Q_k B_k^T \quad (19) \\ \Sigma_{k+1|k+1} &= (1 + \mu_1) (I - K_{k+1} \bar{Y}_{k+1} C_{k+1}) \Sigma_{k+1|k} \end{aligned}$$

$$\begin{aligned} &(I - K_{k+1} \bar{Y}_{k+1} C_{k+1})^T + \\ &(1 + \mu_1^{-1} + \mu_2) \theta K_{k+1} K_{k+1}^T + K_{k+1} \bar{\lambda}_{k+1} (1 - \bar{\lambda}_{k+1}) \\ &l^2 \text{tr}(\bar{X}_{k+1}) \times \\ &K_{k+1} \bar{H}_{k+1} C_{k+1} C_{k+1}^T \bar{H}_{k+1}^T K_{k+1}^T + (1 + \mu_2^{-1}) \bar{\lambda}_{k+1} \\ &K_{k+1} \bar{H}_{k+1} R_{k+1} \bar{H}_{k+1}^T K_{k+1}^T \quad (20) \end{aligned}$$

在初始条件 $\Sigma_{0|0} = P_{0|0} > 0$ 下有正定解 $\Sigma_{k+1|k}$ 和 $\Sigma_{k+1|k+1}$, 则矩阵 $\Sigma_{k+1|k+1}$ 是 $P_{k+1|k+1}$ 的上界。

证明: 对定理1式(17)中的交叉项 A_2 应用引理1,可得

$$A_2 + A_2^T \leq \eta \bar{A}_k P_{k|k} \bar{A}_k^T + \eta^{-1} \bar{\alpha}_k^2 \{[f(x_k) - f(\hat{x}_{k|k})] [f(x_k) - f(\hat{x}_{k|k})]^T\} \quad (21)$$

由式(17)可得

$$\begin{aligned} P_{k+1|k} &\leq (1 + \eta) \bar{A}_k P_{k|k} \bar{A}_k^T + (1 + \eta^{-1}) \bar{\alpha}_k^2 \times \\ &\{[f(x_k) - f(\hat{x}_{k|k})] [f(x_k) - f(\hat{x}_{k|k})]^T\} + \\ &\bar{\alpha}_k(1 - \bar{\alpha}_k) \{f(x_k) f^T(x_k)\} + B_k Q_k B_k^T \quad (22) \end{aligned}$$

由于

$$\begin{aligned} &[f(x_k) - f(\hat{x}_{k|k})] [f(x_k) - f(\hat{x}_{k|k})]^T \leq \\ &\|f(x_k) - f(\hat{x}_{k|k})\|^2 I = \\ &[f(x_k) - f(\hat{x}_{k|k})]^T [f(x_k) - f(\hat{x}_{k|k})] I \quad (23) \end{aligned}$$

故而

$$\begin{aligned} &\{[f(x_k) - f(\hat{x}_{k|k})]^T [f(x_k) - f(\hat{x}_{k|k})]\} \leq \\ &l^2 \{\bar{x}_{k|k} \bar{x}_{k|k}^T\} = l^2 \text{tr}(P_{k|k}) I \quad (24) \end{aligned}$$

将上式代入式(22)得

$$\begin{aligned} P_{k+1|k} &\leq (1 + \eta) \bar{A}_k P_{k|k} \bar{A}_k^T + (1 + \eta^{-1}) \bar{\alpha}_k^2 l^2 \text{tr}(P_{k|k}) I + \\ &\bar{\alpha}_k(1 - \bar{\alpha}_k) l^2 \text{tr}(X_k) I + B_k Q_k B_k^T \quad (25) \end{aligned}$$

同样地,对于定理2式(18)中的交叉项 A_3 和 A_4 ,应用引理2可得

$$\begin{aligned} A_3 + A_3^T &\leq \mu_1 (I - K_{k+1} \bar{Y}_{k+1} C_{k+1}) P_{k+1|k} (I - K_{k+1} \bar{Y}_{k+1} C_{k+1})^T + \\ &\mu_1^{-1} K_{k+1} \{(\bar{y}_{k+1} - y_{k+1})(\bar{y}_{k+1} - y_{k+1})^T\} K_{k+1}^T \quad (26) \end{aligned}$$

$$\begin{aligned} A_4 + A_4^T &\leq \mu_2 K_{k+1} \{(\bar{y}_{k+1} - y_{k+1})(\bar{y}_{k+1} - y_{k+1})^T\} K_{k+1}^T + \\ &\mu_2^{-1} K_{k+1} \{Y_{k+1} \nu_{k+1} \nu_{k+1}^T Y_{k+1}\} K_{k+1}^T \quad (27) \end{aligned}$$

将上述两式代入(18)中,得

$$\begin{aligned} P_{k+1|k+1} &\leq (1 + \mu_1) (I - K_{k+1} \bar{Y}_{k+1} C_{k+1}) P_{k+1|k} (I - K_{k+1} \bar{Y}_{k+1} C_{k+1})^T + \\ &(1 + \mu_1^{-1} + \mu_2) K_{k+1} \{(\bar{y}_{k+1} - y_{k+1})(\bar{y}_{k+1} - y_{k+1})^T\} K_{k+1}^T + \end{aligned}$$

$$K_{k+1} \{ (\bar{Y}_{k+1} C_{k+1} x_{k+1}) (\bar{Y}_{k+1} C_{k+1} x_{k+1})^T \} \times \\ K_{k+1}^T + (1 + \mu_2^{-1}) K_{k+1} \{ Y_{k+1} v_{k+1} v_{k+1}^T Y_{k+1}^T \} K_{k+1}^T \quad (28)$$

考虑到事件触发表达式(5),上式的第二项可进行如下处理

$$K_{k+1} \{ (\bar{y}_{k+1} - y_{k+1}) (\bar{y}_{k+1} - y_{k+1})^T \} K_{k+1}^T \leqslant \theta K_{k+1} K_{k+1}^T \quad (29)$$

将引理3应用到式(28)的后两项:

$$K_{k+1} \{ \bar{Y}_{k+1} C_{k+1} x_{k+1} x_{k+1}^T C_{k+1}^T \bar{Y}_{k+1}^T \} K_{k+1}^T \leqslant \bar{\lambda}_{k+1} (1 - \bar{\lambda}_{k+1}) l^2 \text{tr}(X_{k+1}) K_{k+1} \bar{H}_{k+1} C_{k+1} C_{k+1}^T \bar{H}_{k+1}^T K_{k+1}^T \times \\ \bar{H}_{k+1}^T K_{k+1}^T \quad (30)$$

$$K_{k+1} \{ Y_{k+1} v_{k+1} v_{k+1}^T Y_{k+1}^T \} K_{k+1}^T \leqslant \bar{\lambda}_{k+1} K_{k+1} \bar{H}_{k+1} R_{k+1} \bar{H}_{k+1}^T K_{k+1}^T \quad (31)$$

其中 $\bar{H}_{k+1} = [I, -I]$ 。

将式(29)~(31)代入式(28),有

$$P_{k+1|k+1} \leqslant (1 + \mu_1) (I - K_{k+1} \bar{Y}_{k+1} C_{k+1}) P_{k+1|k} (I - K_{k+1} \bar{Y}_{k+1} C_{k+1})^T + \\ (1 + \mu_1^{-1} + \mu_2) \theta K_{k+1} K_{k+1}^T + \bar{\lambda}_{k+1} (1 - \bar{\lambda}_{k+1}) \times \\ l^2 \text{tr}(X_{k+1}) K_{k+1} \bar{H}_{k+1} C_{k+1} C_{k+1}^T \bar{H}_{k+1}^T K_{k+1}^T + \\ (1 + \mu_2^{-1}) \bar{\lambda}_{k+1} K_{k+1} \bar{H}_{k+1} R_{k+1} \bar{H}_{k+1}^T K_{k+1}^T \quad (32)$$

定理3证毕。

定理4 如果滤波估计增益按如下形式给出,则滤波误差协方差矩阵上界 $\Sigma_{k+1|k+1}$ 的迹可达到最小。

$$K_{k+1} = (1 + \mu_1) \Sigma_{k+1|k} C_{k+1} \bar{Y}_{k+1} \{ \Psi_{k+1} \}^{-1} \quad (33)$$

其中

$$\Psi_{k+1} = (1 + \mu_1) \bar{Y}_{k+1} C_{k+1} \Sigma_{k+1|k} C_{k+1}^T \bar{Y}_{k+1}^T + \\ (1 + \mu_1^{-1} + \mu_2) \theta I +$$

$$\bar{\lambda}_{k+1} (1 - \bar{\lambda}_{k+1}) l^2 \text{tr}(X_{k+1}) \bar{H}_{k+1} C_{k+1} C_{k+1}^T \bar{H}_{k+1}^T + \\ (1 + \mu_2^{-1}) \bar{\lambda}_{k+1} \bar{H}_{k+1} R_{k+1} \bar{H}_{k+1}^T$$

证明:由式(20)中可知 $\Sigma_{k+1|k+1}$

$$\Sigma_{k+1|k+1} = (1 + \mu_1) (I - K_{k+1} \bar{Y}_{k+1} C_{k+1}) \Sigma_{k+1|k} (I - K_{k+1} \bar{Y}_{k+1} C_{k+1})^T +$$

$$(1 + \mu_1^{-1} + \mu_2) \theta K_{k+1} K_{k+1}^T + K_{k+1} \bar{\lambda}_{k+1} (1 - \bar{\lambda}_{k+1}) l^2 \text{tr}(X_{k+1}) K_{k+1} \bar{H}_{k+1} C_{k+1} C_{k+1}^T \bar{H}_{k+1}^T K_{k+1}^T + \\ (1 + \mu_2^{-1}) \bar{\lambda}_{k+1} K_{k+1} \bar{H}_{k+1} R_{k+1} \bar{H}_{k+1}^T K_{k+1}^T$$

为了获得滤波误差协方差矩阵上界 $\Sigma_{k+1|k+1}$ 的最小迹,对式(20)中 $\Sigma_{k+1|k+1}$ 求偏导,并根据引理1可得:

$$\frac{\partial \text{tr}(\Sigma_{k+1|k+1})}{\partial K_{k+1}} = -2(1 + \mu_1) (I - K_{k+1} \bar{Y}_{k+1} C_{k+1}) \Sigma_{k+1|k} C_{k+1}^T \bar{Y}_{k+1}^T \times \\ C_{k+1} \Sigma_{k+1|k} C_{k+1}^T \bar{Y}_{k+1}^T + 2K_{k+1} \{ \Psi_{k+1} \} \quad (34)$$

令 $\frac{\partial \Sigma_{k+1|k+1}}{\partial K_{k+1}} = 0$, 可得 $K_{k+1} = (1 + \mu_1) \Sigma_{k+1|k} C_{k+1} \times$

$\bar{Y}_{k+1} \{ \Psi_{k+1} \}^{-1}$ 。

根据上述定理结果和构造的时变滤波器,将求解时变离散系统滤波算法概括如下:

步骤1 设初始时刻 $k=0$, 给定一些必要的初始条件与信息

步骤2 根据式(6)计算一步预测 $\hat{x}_{k+1|k}$

步骤3 根据式(10)与式(19)计算 $\bar{X}_{k+1}$ 和 $\Sigma_{k+1|k}$

步骤4 设计滤波器增益矩阵 K_{k+1}

步骤5 计算状态估计 $\hat{x}_{k+1|k+1}$

步骤6 计算滤波误差协方差矩阵的上界 $\Sigma_{k+1|k+1}$

步骤7 令 $k=k+1$, 继续执行步骤2

上述算法具有如下优点:①状态估计法包含预测与估计,具有一定的纠错能力;②在估计过程中使用可用的随机非线性、一步测量时滞与事件触发协议等信息;③状态估计具有递推方法,可利用于在线实现。

3 算例仿真

在本部分中,给出算例仿真来说明本文所提出的算法的有效性。

系统参数取值如下:

$$\bar{A}_k = \begin{bmatrix} 0.8 & 0.5 \\ -0.1 & 0.6 + 0.03 \sin(2k) \end{bmatrix}, \bar{B}_k = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}, \\ \bar{C}_k = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

选取非线性函数:

$$\bar{f}(\vec{x}_k) = \begin{bmatrix} 0.72 & 0.3 \\ 0.48 & 0.5 \end{bmatrix} \begin{bmatrix} \vec{x}_{1,k} \\ \vec{x}_{2,k} \end{bmatrix} + \begin{bmatrix} 0.3 \sin(\vec{x}_{1,k} + \vec{x}_{2,k}) \\ 0.1 \vec{x}_{1,k} \sin(2k) \end{bmatrix}$$

此外,其他参数的选取如下:

$$\varepsilon = 0.1, \mu_1 = \mu_2 = 1, \bar{Q}_k = 0.36, \bar{R}_k = 0.5, \bar{\alpha}_k = 0.85, \bar{\lambda}_k = 0.65。$$

在本部分仿真实验中,选取系统的状态初始值为 $x_{0|0} = [0.2 \quad 0.2]^T$, 滤波器的初始值为 $\hat{x}_{0|0} = [0.6 \quad 0.6]^T$ 。系统状态的协方差矩阵的初始值为 $\bar{X}_0 = 2I_2$, 估计误差协方差矩阵的初始值为 $\Sigma_{0|0} = 10I_2$ 。

在进行 MATLAB 算例仿真时,考虑如下两种情形:情形I,当触发阈值 θ 取值为0.1时,给出系统的状态轨迹与滤波器的估计效果对比图;情形II,当触

发阈值 θ 取值为 0.7 时, 给出系统的状态轨迹与滤波器的估计效果对比图。最后, 基于情形 I 和情形 II, 给出系统状态与其估计误差协方差矩阵上界的关系图, MSE1 表示 $\hat{x}_{1,k|k}$ 均方误差, MSE2 表示 $\hat{x}_{2,k|k}$ 均方误差。具体仿真效果图如下:

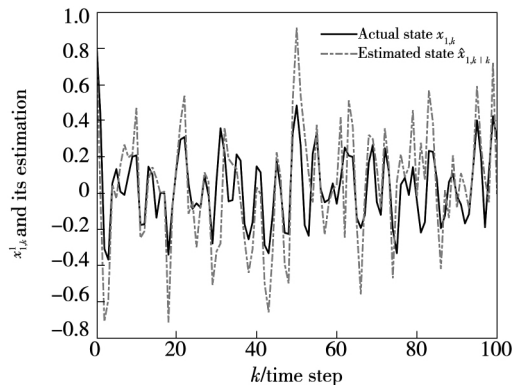


图1 状态 $x_{1,k}$ 及其估计 $\hat{x}_{1,k|k}$ 的轨迹

Fig. 1 The trajectories of state $x_{1,k}$ and its estimation $\hat{x}_{1,k|k}$

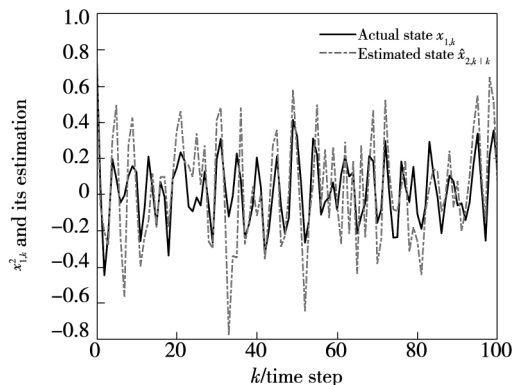


图2 状态 $x_{2,k}$ 及其估计 $\hat{x}_{2,k|k}$ 的轨迹

Fig. 2 The trajectories of state $x_{2,k}$ and its estimation $\hat{x}_{2,k|k}$

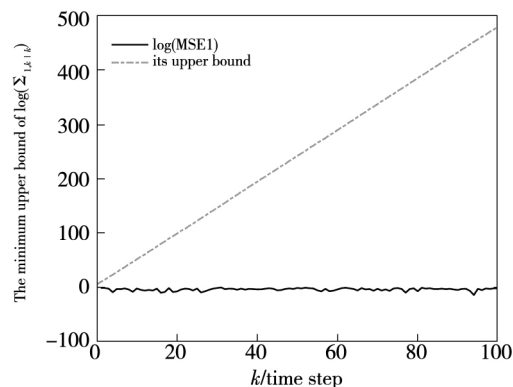


图3 $\log(\text{MSE1})$ 及其上界

Fig. 3 $\log(\text{MSE1})$ and its upper bound

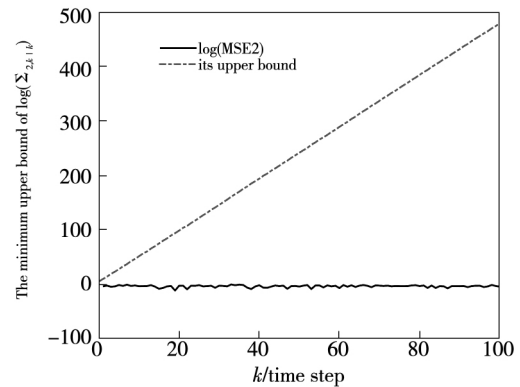


图4 $\log(\text{MSE2})$ 及其上界

Fig. 4 $\log(\text{MSE2})$ and its upper bound

4 结 论

本文中, 解决了一类具有随机发生非线性和一步测量时滞的离散时变系统的滤波问题, 为了刻画一步测量时滞与非线性的随机性, 在文中引入两列服从伯努利分布的随机序列。除此之外, 为了减少网络传输压力, 节省网络带宽资源, 引入了事件触发传输机制。通过求解类黎卡提差分方程, 得到滤波误差协方差矩阵的上界, 并且通过设计相应的滤波增益矩阵使得该上界的迹达到最小。

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