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低采样率下对高频信号的识别方法

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一、问题的提出

对于航空涡喷发动机等旋转机械，其叶片振动信号的提取已由接触式发展到非接触式。转子每转一周，每个叶片经过探头位置一次，产生一次采样。所以探头对单个叶片信号的平均采样率等于转子的运转速率。而根据采样定理[1]，截止频率（Nyguist频率）f_c为采样频率f_s的一半($f_c=f_s/2$)。

一般说来，叶片的各阶固有频率大多高于其运转频率，也即高于截止频率，从而在数据中产生低频和高频分量的混淆问题[2]。在对通常的动态数据进行测量时，人们往往选择采样时间间隔（$t_s=1/f_s$）足够小，以满足采样定理。但在有些情况f_s受转速限制，无法提高。在采样以前先行滤波也不可能，因为传感器接收的原始信号是离散的脉冲序列。为此，我们对于低采样率下识别高频信号的问题进行了方法性的探讨。

二、三次采样的频谱比较法

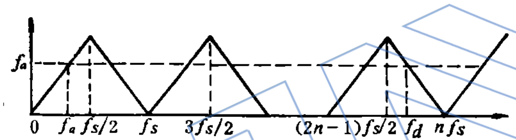

图1 数据频率的折叠形式

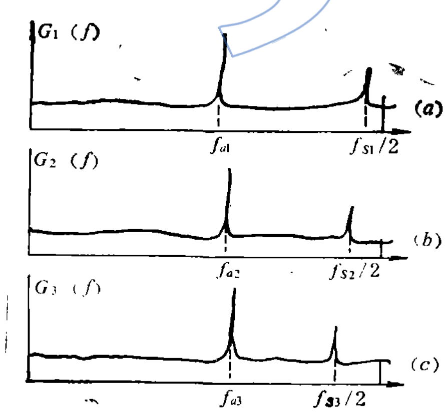

图2 三种采样率下的频谱图

如图1所示，原始数据的真实频率f_d（设$f_d>f_s/2$）被误为（0，$f_s/2$)频率范围内的差频f_a，其实还混有高频。

即$f_d=nf_s\pm f_a$，而n为不能预知的某正整数。

对于一般类型的动态数据，f_d是恒定的。而旋转着的叶片，由于离心力的影响，其自振频率f_d却随转速f_s变化。f_d比非运转状态的自振频率f_0高，即$f_d^2=f_0^2+\alpha f_s^2$

α为无量纲常数，与叶片及盘的几何参数有关。由此，得

$$nf_s\pm f_a=\sqrt{f_0^2+\alpha f_s^2} \tag{1}$$

对于三种f_{s1}，f_{s2}和f_{s3}测得的振动信号作频谱分析，得图2的频响函数。对应的差频f_{a1}、f_{a2}和f_{a3}，应满足(1)，则：式，

$$\left.\begin{aligned} f_{d1}&=nf_{s1}\pm f_{a1}=\sqrt{f_0^2+\alpha f_{s1}^2}\\ f_{d2}&=nf_{s2}\pm f_{a2}=\sqrt{f_0^2+\alpha f_{s2}^2}\\ f_{d3}&=nf_{s3}\pm f_{a3}=\sqrt{f_0^2+\alpha f_{s3}^2}\end{aligned}\right\} \tag{2}$$

这里f_{s1}、f_{s2}和f_{s3}之差较小，不致引起n的变化。消去(2)式中f_0和α，解得倍数n：

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$$n=\pm\frac{1}{2}\;\frac{f^2_{a1}(f^2_{s3}-f^2_{s2})-f^2_{a2}(f^2_{s3}-f^2_{s1})+f^2_{a3}(f^2_{s2}-f^2_{s1})}{-f_{s1}f_{a1}(f^2_{s3}-f^2_{s2})+f_{s2}f_{a2}(f^2_{s3}-f^2_{s1})-f_{s3}f_{a3}(f^2_{s2}-f^2_{s1})} \tag{3}$$

通常我们取$f_{s1}<f_{s2}<f_{s3}$，或$f_{s1}>f_{s2}>f_{s3}$，即 $(f_{s2}-f_{s1})(f_{s3}-f_{s2})>0$

为保证f_{d1}、f_{d2}、f_{d3}及n的值为非负，用符号函数：

$$\mathop{\mathrm{sgn}}_{(i,j=1,2,3,i\neq j)}[(f_{si}-f_{sj})(f_{aj}-f_{ai})]\begin{cases}1 & (f_{si}-f_{sj})(f_{aj}-f_{ai})>0\\ 0 & (f_{si}-f_{sj})(f_{aj}-f_{ai})=0\\ -1 & (f_{si}-f_{sj})(f_{aj}-f_{ai})<0\end{cases} \tag{4}$$

按（3）式求n值取整后代回（2）式，可求得

$$f_{di}=nf_{si}\pm f_{ai} \qquad (i=1,2,3) \tag{5}$$

三、识别方法的误差分析

如图 2 频谱峰不太密集的情况，很容易找出图中各峰的对应关系。对于同一阶自振频率，折叠后相应的峰值应该大致相等，如图中 $H_1\approx H_2\approx H_3$，据此确定 f_{a1}、f_{a2} 和f_{a3} 的位置。因为n是整数序列、阶跃变化的，可使三次采样维持n不变。也就是保证f_{d1}、f_{d2}和f_{d3}在图 1 的同一直线段内，不越过转折点。此时采集是有效的，可证明其必要条件是

$$(f_{a2}-f_{a1})(f_{a3}-f_{a2})>0 \tag{6}$$

如需要，还可以求出α和f_0，

$$\alpha=(f^2_{di}-f^2_{di})/(f^2_{di}-f^2_{dj}) \qquad (i\neq j) \tag{7}$$

$$f_0=\sqrt{f^2_{di}-\alpha f^2_{si}} \qquad (i=1,2,3) \tag{8}$$

对f_{s1}、f_{s2}和f_{s3}的测量一般较准确，必须考虑的是f_{a1}、f_{a2}和f_{a3}的测量误差，它是由频率分辨力等多方面因素引起的，记各量的相对误差皆为δ_a，由此n产生的相对误差为

$$\delta_n=\sqrt{\beta^2_{a1}+\beta^2_{a2}+\beta^2_{a3}}\;\delta_a,$$

而$\beta_{ai}=(f_{ai}/n)(\partial n/\partial\alpha_i)$　$(i=1,2,3)$，所以

$$\left.\begin{aligned}\beta_{a1}&=f_{d1}f_{a1}(f^2_{s3}-f^2_{s2})/\Delta\\ \beta_{a2}&=-f_{d2}f_{a2}(f^2_{s3}-f^2_{s1})/\Delta\\ \beta_{a3}&=f_{d3}f_{a3}(f^2_{s2}-f^2_{s1})/\Delta\end{aligned}\right\} \tag{9}$$

其中$\Delta=\pm[f^2_{a1}(f^2_{s3}-f^2_{s2})-f^2_{a2}(f^2_{s3}-f^2_{s1})+f^2_{a3}(f^2_{s2}-f^2_{s1})]/2$。

$$\delta_n=\frac{2\delta_a\sqrt{f^2_{a1}(f^2_{s3}-f^2_{s2})^2+f^2_{a2}(f^2_{s3}-f^2_{s1})^2+f^2_{s2}(f^2_{s2}-f^2_{s1})^2}}{|f^2_{a1}(f^2_{s3}-f^2_{s2})-f^2_{a2}(f^2_{s3}-f^2_{s1})+f^2_{a3}(f^2_{s2}-f^2_{s1})|} \tag{10}$$

由测量误差引起f_{d1}、f_{d2}和f_{d3}的相对误差与δ_n在同一量级。所以要使误差较小，应保持f_{s1}、f_{s3}和f_{s2}之间有较大的差值，以避免f_{a1}、f_{a2}和f_{a3}间在带权加减运算时损失有效数字。

四、算例及讨论

我们取一个算例：某叶片根部安装角为10°，截面面积5.0cm²；尖部安装角为60°，截面面积2.5cm²。轮盘外半径$r_0=15$cm，叶尖距盘心R＝31cm。测得它第一阶弯曲静频$f_0=$180Hz，第二阶弯曲静频$f'_0=520$Hz。由已知几何参数经查表得到B＝2.13，B′＝13.13。

根据常用经验公式$f_{di}=\sqrt{f^2_0+B(N_{si}/60)^2}$得到在三种转速：$N_{s1}=11820$，$N_{s2}=$

12000，$N_{s3}=12120r/min$时的自振频率为$f_{d1}=339.2096$，$f_{d2}=342.9286$，$f_{d3}=345.4164(Hz)$。另由$f'_{di}=\sqrt{f'_0+B'(N_{si}/60)^2}$，分别得到$f'_{d1}=883.1547$，$f'_{d2}=891.9641$，$f'_{d3}=897.8622Hz$。

为考察本文提出的三次采样谱图比较法，以一个模拟试验对照上例。无须预知f_0(或f'_0)及B(或B′)，只要在三种不同的采样频率$f_{si}(=N_{si}/60)$：$f_{s1}=197$，$f_{s2}=200$，$f_{s3}=202Hz$进行采样，得到类似图2的三条频谱曲线（在$0\sim f_{si}/2$范围内），找到相应的峰值位置：$f_{a1}=54.7904$，$f_{a2}=57.0714$，$f_{a3}=58.5836Hz$和$f'_{a1}=95.1547$，$f'_{a2}=91.9641$，$f'_{a3}=89.8622Hz$。

当$f_{s1}<f_{s2}<f_{s3}$，有$f_{a1}<f_{a2}<f_{a3}$，所以在式(6)中取负号，算得$n=1.9387$；$f'_{a1}>f'_{a2}>f'_{a3}$，在式(6)中取正号，算得$n'=4.0233$。将此两个倍数取整后得：$n=2$，$n'=4$。利用(5)式，有

$$f_{di}=nf_{si}-f_{ai}\qquad(i=1,2,3)\tag{11}$$

$$f'_{di}=n'f_{si}+f'_{ai}\qquad(i=1,2,3)\tag{12}$$

将f_{si}、f_{ai}和f'_{ai}代入以上式，得到结果与前述完全相符。

本文方法考虑了原始信号本身随采样频率的变化因素，具有一般推广性。McCarty[4]等由于忽视了$\alpha>0$这一因素，引起了较大的误差。而在原来计算转子叶片自振频率的经验公式中，参数α的确定是很困难的，本文却能非常容易地反推出此值。根据α值可以进一步区分叶片振动的弯曲和扭转形式。并且文中的计算公式从理论上是完全成立的，只是要求每次频谱的测量保证有较高的精确性。

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OPTIMAL CONTROL OF A MARINE GAS TURBINE PROPULSION SYSTEM

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Abstract

A design method of"Linear Quadratic Output PI Feedback Control" is proposed in this paper. Amarine gas turbine propulsion system controller has been designed by use of this method. The controller not only has the advantages of the PI controller in classical control theory, but also those of the state-variable feedback in modern control ʇheory. The simulation of this controller gives satisfactory results.

TECHNICAL NOTES

A CONTOUR LINE PLOTTING SYSTEM WITH POLAR COORDINATES FOR AEROENGINE INLET FLOW FIELD

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Abstract

Three interpolating surface algorithms, i.e. bilinear, biquadratic and circular spline function radial parabolic algorithms, are provided in this programming system. The last of them is considered to be the most ideal one for interpolating surface because it has high accuracy, truly expresses the distorted flow field, and overcomes the unsmoothed joints of the biquadratic algorithm. In order to determine the intersection points between contour and network, rectangular grids of polar coordinates instead of rectangular coordinates are used. In result, many advantages for engine inlet flow field plotting can thus be achieved. This programming system can save computer time and reduce costs.

IDENTIFICATION OF HIGH FREQUENCY SIGNAL UNDER LOW SAMPLING RATIO

Zhang Yisong and Yi Liyan

(*Nanjing Aeronautical Institute*)

Abstract

Considering that the highest resolving frequncy in dynamic data is restricted by sampling ratio, a new method is proposed to identify the high frequency signal from three low-ratio sampling spectra. The complicated case of vibration measurement of

rotary blades is investigated with error analysis and illustratrated by a numerical example.

DYNAMIC FLEXIBILITY COEFFICIENT MATRIX AND ITS MEASUREMENT FOR AEROENGINE SUPPORTING SYSTEM

Li Zhiwei and Yang Shenji
(Northwestern polytechnical University)

Abstract

This paper deals with the concept of the dynamic flexibility coefficient matrix and the measuremnt of the coefficients. As an example, the method of the coefficient measurement and the results of the coefficient matrix calculation are given for the supporting system of a twin-spool jet engine. The method and results will be useful for practice.

EQUIVALENT TEST OF PLUNGER FUEL PUMP

Zou Pixian, Xue Jianhua, He Xiude
(Far East Machinery Company)

Abstract

The equivalent test presented in this paper is a technique developed to accelerate the life test. It is achieved by increasing the testing load whereas reducing the test time to make the sample product being tested have approximately the same degree of damage as compared with the main failure mode. The epuivalent formula for this purpose has been deduced from analysis and calculation, and the equivalent test specifications can be thus compiled on the basis of the epuivalent formula.

MEASUREMENT OF RESIDENCE TIME DISTRIBUTION FUNCTIONS OF FUEL IN COMBUSTOR

Lu Xinli and Chen Jie
(Northwestern Polytechnical University)

Abstract

An experimental epuipment for measuring the residence time distribution functions of fuel in combustor has been made, based on the absorption of radiation from a line source at 253.7nm generated by mercury vapour. The residence time distribution functions of fuel both in a main conbustor and a model afterburner under cool conditions have been measured by means of this equipment and have been analyzed.