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旋转叶片上的三维附面层

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一、方程及数学处理

在附壁的非正交曲线坐标系下，见图1。张量形式的流体运动方程见参考文献〔1〕。

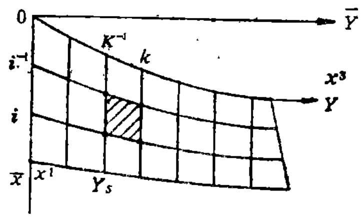

图1 坐标系

我们假设：（1）流体呈定常运动；（2）忽略彻体力；（3）TSL条件成立。在基本方程中，引入逆变物理分量代替逆变分量，并根据假设进行简化和量级分析，得到一组普遍适用的附面层方程：

$$\frac{\partial(\rho W^1\sqrt{g_{33}}\sin\theta)}{\partial x^1}+\frac{\partial(\rho W^2\sqrt{g_{11}g_{33}}\sin\theta)}{\partial x^2}+\frac{\partial(\rho W^3\sqrt{g_{11}}\sin\theta)}{\partial x^3}=0 \quad (1)$$

$$\frac{W^1}{\sqrt{g_{11}}}\frac{\partial W^1}{\sqrt{\partial x'}}\frac{W^2}{\sqrt{g_{22}}}\frac{\partial W^1}{\partial x^2}+\frac{W^3}{\sqrt{g_{33}}}\frac{\partial W^1}{\partial x^3}-K_1\frac{\cos\theta}{\sin\theta}\cdot(W^1)^2+k_2\cdot\frac{1}{\sin\theta}(W^3)^2$$

$$+K_{12}W^1W^3-\frac{\omega^2R}{\sin^2\theta}[\cos(\vec{R},\vec{e}_1)-\cos(\vec{R},\vec{e}_3)\cos\theta]+\frac{2\omega\cos(\vec{\omega},\vec{e}_2)}{\sin\theta}(\cos\theta W^1+W^3)$$

$$\frac{2\omega\cos(\vec{\omega},\vec{e}_3)}{\sin\theta}=W^2-\frac{1}{\rho\sin^2\theta\sqrt{g_{11}}}\cdot\frac{\partial P}{\partial x^1}+\frac{\cos\theta}{\rho\sin^2\theta\sqrt{g_{33}}}\cdot\frac{\partial P}{\partial X^3}$$

$$+\frac{1}{\rho}\cdot\frac{\partial}{\partial x^2}\left[\mu\cdot\frac{\partial W^1}{\partial x^2}-\mu\cos\theta\frac{\partial W^3}{\partial x^2}-\rho\overline{(W^1W^2)^1}\right] \quad (2)$$

$$-\frac{\partial P}{\partial x^2}=-\omega^2R\cos(\vec{R},\vec{e}_2)+2\omega\cos(\vec{\omega},\vec{e}_3)\frac{W^1+\cos\theta W^3}{\sin\theta}-2\omega\cos(\vec{\omega},\vec{e}_1)\frac{\cos\theta W^1+W^3}{\sin\theta} \quad (3)$$

$$\frac{W^1}{\sqrt{g_{11}}}\cdot\frac{\partial W^3}{\partial x^1}+\frac{W^2}{\sqrt{g_{22}}}\cdot\frac{\partial W^3}{\partial x^2}+\frac{W^3}{\sqrt{g_{33}}}\cdot\frac{\partial W^3}{\partial x^3}-K_2\frac{\cos\theta}{\sin\theta}(W^3)^2+K_1\frac{1}{\sin\theta}(W^1)^2$$

$$+K_{21}W^1W^3-\frac{\omega^2R}{\sin^2\theta}[\cos(\vec{R},\vec{e}_3)-\cos(\vec{R},\vec{e}_1)\cos\theta]$$

$$+\frac{2\omega}{\sin\theta}\cos(\vec{\omega},\vec{e}_1)W^2-\frac{2\omega}{\sin\theta}\cos(\vec{\omega},\vec{e}_2)(\cos\theta W^3+W^1)$$

$$=\frac{\cos\theta}{\rho\sin^2\theta\sqrt{g_{11}}}\cdot\frac{\partial P}{\partial x^1}-\frac{1}{P\sin^2\theta\sqrt{g_{33}}}\cdot\frac{\partial P}{\partial x^3}$$

$$+\frac{1}{P}\cdot\frac{\partial}{\partial x^2}\left[\mu\frac{\partial W^3}{\partial x^2}-\mu\cos\theta\frac{\partial W^1}{\partial x^2}-\rho\overline{(W^3W^2)^1}\right] \quad (4)$$

由量级分析可知，在一般旋转速度场中$\partial p/\partial x^1$与$\partial p/\partial x^2$有大致相同的量级，因而不能令

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$\partial p/\partial x^2=0$。目前兴起的几种处理$\partial p/\partial x^2\neq 0$的方法对三维情况都很复杂。做为发展程序的第一步，也为和现有的典型的三维结果相比较，本文只处理$\partial p/\partial x^2=0$的情况，这也适用于$\omega\sim(U/L)(\delta/L)$的情况。U为速度尺度。

故设$(\partial p/\partial x^2)=0$，对应方程组(1)、(2)、(4)、(5)的边界条件是：$x^2=0$时，$W^1=W^3=0$，$x^2=\delta$时，$W^1=W^1_e(x^1,x^3)$，$W^3=W^3_e(x^1,x^3)$

对上述方程采用与文献〔2〕相似的处理方法，但作以下改进。首先利用方程(5)以及附面层外缘的特殊情况，可用速度分布取代各方程中的压力项。经过Folknar-Sken变换，降阶处理，在数值离散中引入两个方向的加权因子，γ和ϕ，消除数值振动，最后进行线性化。

把最终的差分方程分成两个部分，它们有共同的未知变量。分别求解，再交叉迭代。在叶根处，对附面层方程进行消除奇异性处理，得到附着线方程。其求解与上述过程类似。

二、算例与结果分析

按上述思想编为程序。算例是平板上面放有一个柱体(图2)。无粘流场由下面的关系给出：

$$u_e=u_\infty[1+\alpha^2(\Delta_2/\Delta_1{}^2)],\ W_e=-2U_\infty a^2(\Delta_3/\Delta^2{}_1)$$

其中$\Delta_1=(x-x_0)^2+Z^2$，$\Delta_2=-(x-x_0)^2+Z^2$，

$\Delta_3=(x-x_0)Z$，a是圆柱半径，U_∞是参考速度。

给定$u_\infty=3050$cm/s，$a=6.1$cm，$x_0=47.5$cm，$g_\infty=8.0$沿y方向取11个点，$\Delta y=0.8$、$\Delta x=1.22$cm、$\Delta Z=0.61$cm。对比结果见表1。从表1可见，三组结果非常接近，最大相对误差为10^{-3}，它表明，本方法和程序具有一定的工程实用价值。本方法已用于平板层流附面层计算。本方法的优点：(1)可处理非常复杂的壁面情况；(2)可以处理带有旋转的叶面附面层；(3)收敛快，耗时少，并且稳定性好。

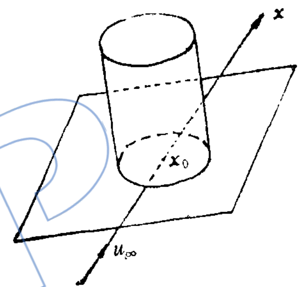

图2 算例示意图

表1 计算结果对比

X (cm)	f_w'' (Z = 0)			f_w'' (Z = 0.61cm)		
	本方法	文献〔2〕	文献〔3〕	本方法	文献〔2〕	文献〔3〕
1.22	0.329498	0.329498	0.329498	0.329461	0.329461	0.329461
2.44	0.327975	0.327973	0.327973	0.327894	0.327711	0.327712
3.66	0.326239	0.326233	0.326233	0.326104	0.325852	0.325853
4.88	0.324263	0.324252	0.324251	0.324063	0.323584	0.323588
6.10	0.322004	0.321987	0.321985	0.321725	0.321103	0.321109
7.32	0.319412	0.319416	0.319385	0.319037	0.318078	0.318112

参考文献

〔1〕汪庆桓、陈乃兴，"非正交曲线坐标系在叶轮机械粘性流动计算中的应用"，工程热物理学报，vol.2，No.4，1981。

〔2〕Tuncer Cebeci, Kalle Kaups and Juolg A.Ramseg, "A General Method for Calculating 3-D Compressible Laminar and Turbulant Boundary Layers on Arbitrary Wings",NASA CR 2777.

〔3〕Tuncer Cebeci, Kalle Kaups abd Aefred Moser, "Calculation of 3-D Boundary Layers Ⅱ, 3-D Flows in Cartesion Coordinates", AIAA J.VoL.13, Aug.1975.

EFFECTS OF Al_2O_3 DEPOSIT ON PERFORMANCE OF SOLID PROPELLANT ROCKET ENGINE

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Abstract

The Al_2O_3 mass fraction collected from the interior surface of nozzle is correlated with the material of throat insert, combustion pressure and Al content in the propellant. The thrust loss resulted from Al_2O_3 deposition should be considered in performance prediction and has been calculated in this paper.

ESTABLISHMENT OF VARIABLE VANE REGULATION LAW

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Abstrca

An approach to establish optimum regulation law of the variable vane for a turbojet engine is presented. The stable operating boundary of the engine can be determined according to matching with effective flight conditions.

THE 3-D BOUNDARY LAYER ON ROTATING BLADE

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Abstract

The 3-D boundary layer equations with rotation terms are presented in oblique curvilinear coordinates. After treatment of the governing equations, the basic equations are discretized by backwind difference scheme, and the two parts of them are solved by cross iteration.