第 4 卷　第 4 期　　航空动力学报　　Vol. 4　No. 4
1989 年　10 月　　Journal of Aerospace Power　　Oct.　1989

隐式通量分裂线松驰迭代法求解平面叶栅无粘绕流问题

清华大学　黄东涛　沈孟育　　科学院计算中心　张耀科

【摘要】　本文工作是在 MacCormack 通量分裂格式[2]基础上发展的一种有限面积通量分裂隐式格式，其特点是在求解隐式离散化方程时，采用往返扫描一次的 Gauss-Seidel 线松驰迭代方法，避免了对时间步长的任何限制。为提高定常解精度，格式的显式右端项采用二阶精度的离散。在数值求解跨音速涡轮平面叶栅问题中，对壁面边界作了较仔细的隐式处理。数值计算表明本方法保持了 MacCormack 格式具有的高收敛速率，(约 30 个时间步即可达到定常解) 而每一时间步计算量约减少一半。数值结果与实验结果符合得很好。

一、前　　言

在计算流体领域，研究和发展具有高效快速收敛特点的数值方法，一直是个重要的研究课题。近年来发展的矢通量分裂格式由于较好地模拟了物理上扰动信息的传播规律，是一种很有前途的方法。J. L. Steger 和 R. F. Warming 于 1981 年对此方法作了较全面的论述[1]。1985 年 R. W. MacCormack 发展了一种非因式分解的通量分裂格式[2]，本文的工作主要是在后者的基础上进行的，即把通量分裂方法用于有限面积单元，建立有限面积的通量分裂隐式格式，并在求解隐式离散化方程时，采用往返扫描一次的 Gauss-Seidel 线松驰迭代，避免了对时间步长的任何限制。另一方面，考虑到对于通量分裂格式，预测-校正的两步方法并不能提高格式精度，相反在每一个时间步中却增加了近一倍的计算工作量，固而不予采用。在数值模拟跨音速平面叶栅绕流问题中，显式右端项采用了二阶精度的离散，壁面边界作了较精细的分析处理，从而提高了定常解的精度，数值算例表明，我们的这一改进，既保持了通量分裂的 MacCormack 格式具有的高速收敛的优点（约 30 个时间步既可获得定常解)，而每一时间步计算工作量约减少 1/2。同时，由于提高了显式部分的格式精度及壁面边界处理的准确度，使平面叶栅绕的数值结果与实验结果相比较符合得很好。这一方法可进一步发展用于粘流问题的计算。

二、基本出发方程

直角坐标的 Euler 方程无量纲守恒形式为[4]:

$$\partial U/\partial t + \partial F/\partial x + \partial G/\partial y = 0 \qquad (1)$$

其中 $U=(\rho,\ \rho u,\ \rho v,\ E)^T$，$F=(\rho u,\ p+\rho u^2,\ \rho uv,\ (E+p)u)^T$，$G=(\rho v,\ \rho uv,\ P+\rho v^2,\ (E+p)v)^T$，$P$ 为压力，ρ 为密度，u、v 分别为 x 方向和 y 方向的速度分量，$E=P/(\gamma-1)+\rho(u^2+v^2)/2$ 为总能，γ 为气体绝热指数。引入完全气体假设，则无量纲状态方程为:

$$P = \rho T/\gamma \qquad (2)$$

为了采用有限面积法，将 (1) 式在求解域的任意一个面积上积分，并运用 *Gauss* 散度定理:

$$\int_V \frac{\partial U}{\partial t} dV = -\oint_s \vec{P}\cdot d\vec{s} \tag{3}$$

其中 $\vec{P}=F\vec{e}_x+G\vec{e}_y$，$V$ 表示积分域，s 为该域边界。若引入任意的斜交曲线坐标系 $\xi=\xi(x,y)$，$\eta=\eta(x,y)$。并定义 $\vec{S}^{(\xi)}$、$\vec{s}^{(\eta)}$ 分别为 ξ 网格坐标线和 η 网格坐标线的单位法向，即:

$$\vec{S}^{(\xi)}=(s_x^{(\xi)},s_y^{(\xi)})=1/\sqrt{y_\xi^2+x_\xi^2}\,(-y_\xi,x_\xi)$$

$$\vec{s}^{(\eta)}=(s_x^{(\eta)},s_y^{(\eta)})=1/\sqrt{y_\eta^2+x_\eta^2}\,(y_\eta,-x_\eta)$$

这样 (3) 式在任意的曲线坐标系可写成

$$\int_V \frac{\partial U}{\partial t} dV = -\oint_s (F' ds^{(\eta)} - G' ds^{(\xi)}) \tag{4}$$

其中: $F'=s_x^{(\eta)}F+s_y^{(\eta)}G$，$G'=s_x^{(\xi)}F+s_y^{(\xi)}G$ 为曲线坐标系下的矢通量项，$ds^{(\eta)}=\sqrt{y_{(\eta)}^2+x_{(\eta)}^2}\,d\eta$，$ds^{(\xi)}=\sqrt{y_{(\xi)}^2+x_{(\xi)}^2}\,d\xi$ 为相应的坐标弧长。

三、有限面积的矢通量分裂隐式方法

引入:
$$A=\partial F/\partial U,\qquad B=\partial G/\partial U \tag{5}$$

由于 F、G 均为 U 的一次齐次函数，故 $F=AU$，$G=BU$。若令: $A'=s_x^{(\eta)}A+s_y^{(\eta)}B$，$B'=s_x^{(\xi)}A+s_y^{(\xi)}B$。则 $F'=A'U$，$G'=B'U$

对 A'、B' 进行相似变换可得:

$$A'=N^{-1}C_{A'}^{-1}\Lambda_{A'}C_{A'}N,\qquad B'=N^{-1}C_{B'}^{-1}\Lambda_{B'}C_{B'}N \tag{6}$$

其中:

$$\Lambda_{A'}=\begin{bmatrix} u' & & & 0\\ & u' & & \\ & & u'+c & \\ 0 & & & u'-c\end{bmatrix},\qquad \Lambda_{B'}=\begin{bmatrix} v' & & & 0\\ & v' & & \\ & & v'+c & \\ 0 & & & v'-c\end{bmatrix}$$

$$C_{A'}^{-1}=\begin{bmatrix}1 & 0 & 1/2c^2 & 1/2c^2\\ 0 & -s_y^{(\eta)} & s_x^{(\eta)}/2\rho c & -s_x^{(\eta)}/2\rho c\\ 0 & s_x^{(\eta)} & s_y^{(\eta)}/2\rho c & -s_y^{(\eta)}/2\rho c\\ 0 & 0 & 1/2 & 1/2\end{bmatrix},\qquad C_{A'}=\begin{bmatrix}1 & 0 & 0 & -1/c^2\\ 0 & -s_y^{(\eta)} & s_x^{(\eta)} & 0\\ 0 & s_x^{(\eta)}\rho c & s_y^{(\eta)}\rho c & 1\\ 0 & -s_x^{(\eta)}\rho c & -s_y^{(\eta)}\rho c & 1\end{bmatrix}$$

$$C_{B'}^{-1}=\begin{bmatrix}1 & 0 & 1/2c^2 & 1/2c^2\\ 0 & -s_y^{(\xi)} & s_x^{(\xi)}/2\rho c & -s_x^{(\xi)}/2\rho c\\ 0 & s_x^{(\xi)} & s_y^{(\xi)}/2\rho c & -s_y^{(\xi)}/2\rho c\\ 0 & 0 & 1/2 & 1/2\end{bmatrix},\qquad C_{B'}=\begin{bmatrix}1 & 0 & 0 & -1/c^2\\ 0 & -s_y^{(\xi)} & s_x^{(\xi)} & 0\\ 0 & s_x^{(\xi)}\rho c & s_y^{(\xi)}\rho c & 1\\ 0 & -s_x^{(\xi)}\rho c & -s_y^{(\xi)}\rho c & 1\end{bmatrix}$$

$$N=\begin{bmatrix}1 & 0 & 0 & 0\\ -\frac{u}{\rho} & \frac{1}{\rho} & 0 & 0\\ -\frac{v}{\rho} & 0 & 1/\rho & 0\\ (\gamma-1)\alpha & -(\gamma-1)u & -(\gamma-1)v & \gamma-1\end{bmatrix},\qquad N^{-1}=\begin{bmatrix}1 & 0 & 0 & 0\\ u & \rho & 0 & 0\\ v & 0 & \rho & 0\\ \alpha & \rho u & \rho v & 1/(\gamma-1)\end{bmatrix}$$

$$\alpha=(u^2+v^2)/2\ ,\quad u'=s_x^{(\eta)}u+s_y^{(\eta)}v\ ,\quad v'=s_x^{(\xi)}u+s_y^{(\xi)}v$$

若将$\wedge_A$、$\wedge_B$按其特征值符号分解为:

$$\wedge_A'=\wedge_A^{+}{}'+\wedge_A^{-}{}'\ ,\qquad \wedge_B'=\wedge_B^{+}{}'+\wedge_B^{-}{}' \tag{7}$$

相应得:

$$\begin{aligned}A'^{+}&=N^{-1}C_A^{-1}\wedge_A^{+}C_AN & A'^{-}&=N^{-1}C_A^{-1}\wedge_A^{-}C_AN\\ B'^{+}&=N^{-1}C_B^{-1}\wedge_B^{+}C_BN & B'^{-}&=N^{-1}C_B^{-1}\wedge_B^{-}C_BN\end{aligned} \tag{8}$$

定义:

$$F'^{+}=A'^{+}U\ ,\quad F'^{-}=A'^{-}U\ ,\quad G'^{+}=B'^{+}U\ ,\quad G'^{-}=B'^{-}U \tag{9}$$

则

$$F'=F'^{+}+F'^{-}\ ,\qquad G'=G'^{+}+G'^{-} \tag{10}$$

即矢通量项F'、G'按其特征值正负分裂为F'^{+}、F'^{-}和G'^{+}、G'^{-}。

在物理平面上引入(ζ η)坐标网格线, 计算单元为任意的四边形(图 1), 方程(2-4)的积分域即为单元的四边形, 定义:

$$U_{ij}=\frac{1}{V_{ij}}\int_{V_{ij}}U\,dV \tag{11}$$

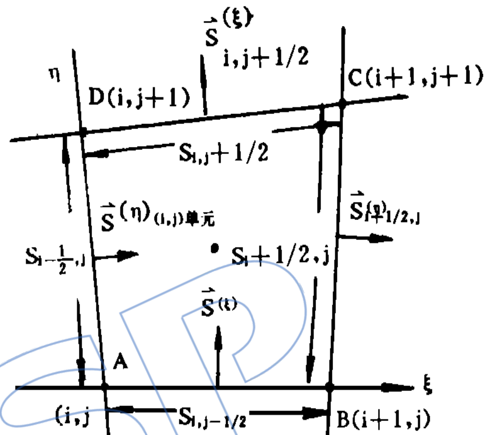

图 1　第(i, j)计算单元的网格节点、边界围线及其法向

若空间只取一阶精度, 则(4)式的显式通量分裂格式写成:

$$\begin{aligned}\Delta U_{ij}^n&=U_{ij}^{n+1}-U_{ij}^n\\ &=-\frac{\Delta t}{V_{ij}}\Big[\big(A'^{+n}_{i+1/2,j}U_{ij}^n+A'^{-n}_{i+1/2,j}U_{i+1,j}^n\big)\cdot s_{i+1/2,j}\\ &\quad-\big(A'^{+n}_{i-1/2,j}U_{i-1,j}^n+A'^{-n}_{i-1/2,j}U_{i,j}^n\big)\cdot s_{i-1/2,j}\\ &\quad+\big(B'^{+n}_{j+1/2,j}U_{i,j}^n+B'^{-n}_{i,j+1/2}U_{i,j+1}^n\big)\cdot s_{i,j+1/2}\\ &\quad-\big(B'^{+n}_{i,j-1/2}U_{i,j-1}^n+B'^{-n}_{i,j-1/2}U_{ij}^n\big)\cdot s_{i,j-1/2}\Big]\end{aligned} \tag{12}$$

而隐式增量写成:

$$\begin{aligned}\delta U_{ij}^n=&-\frac{\Delta t}{V_{ij}}\Big[\big(A'^{+n}_{i+1/2,j}U_{i,j}^{n+1}+A'^{-n}_{i+1/2,j}U_{i+1,j}^{n+1}\big)\cdot s_{i+1/2,j}-\big(A'^{+n}_{i-1/2,j}U_{i-1,j}^{n+1}+A'^{-n}_{i-1/2,j}U_{i,j}^{n+1}\big)\cdot s_{i-1/2,j}\\ &+\big(B'^{+n}_{i,j+1/2}U_{i,j}^{n+1}+B'^{-n}_{i,j+1/2}U_{i,j+1}^{n+1}\big)\cdot s_{i,j+1/2}-\big(B'^{+n}_{i,j-1/2}U_{i,j-1}^{n+1}+B'^{-n}_{i,j-1/2}U_{ij}^{n+1}\big)\cdot s_{i,j-1/2}\Big]\end{aligned} \tag{13}$$

(13)式减去(12)式, 经整理得矢通量分裂隐式计算格式:

$$\hat{E}_{ij}^n\delta U_{i-1,j}^n+\hat{D}_{ij}^n\delta U_{i+1,j}^n+\hat{A}_{ij}^n\delta U_{ij}^n+\hat{C}_{ij}^n\delta U_{i,j-1}^n+\hat{B}_{ij}^n\delta U_{i,j+1}^n=\Delta U_{ij}^n \tag{14}$$

其中: $\hat{E}_{ij}^n=-\Delta t/V_{ij}\cdot A'^{+n}_{i-1/2,j}\cdot S_{i-1/2,j}$, $\hat{D}_{ij}^n=\Delta t/V_{ij}\cdot A'^{-n}_{i+1/2,j}\cdot S_{i+1/2,j}$

$\hat{A}_{ij}^n=I+\Delta t/V_{ij}\cdot\big[A'^{+n}_{i+1/2,j}\cdot S_{i+1/2,j}-A'^{-n}_{i-1/2,j}\cdot S_{i-1/2,j}+B'^{+n}_{i,j+1/2}\cdot S_{i,j+1/2}-B'^{-n}_{i,j-1/2}\cdot S_{i,j-1/2}\big]$

$\hat{C}_{ij}^n=-(\Delta t/V_{ij})\,B'^{+n}_{i,j-1/2}\cdot S_{i,j-1/2}$, $\hat{B}_{ij}^n=(\Delta t/V_{ij})\,B'^{-n}_{i,j+1/2}\cdot S_{i,j+1/2}$

对于定常解, 其精度只由(14)式的右端项ΔU_{ij}^n的离散精度决定, 为此ΔU_{ij}^n取二阶精度为:

$$\begin{aligned}\Delta U_{ij}^n=-\Delta t/V_{ij}\Big[&\big(A'^{+n}_{i+1/2,j}U^{+n}_{i+1/2,j}+A'^{-n}_{i+1/2,j}U^{-n}_{i+1/2,j}\big)\cdot S_{i+1/2,j}\\ &-\big(A'^{+}_{i-1/2,j}U^{+n}_{i-1/2,j}+A'^{-}_{i-1/2,j}U^{-n}_{i-1/2,j}\big)\cdot S_{i-1/2,j}\\ &+\big(B'^{+n}_{i,j+1/2}U^{+n}_{i,j+1/2}+B'^{-n}_{i,j+1/2}U^{-n}_{i,j+1/2}\big)\cdot S_{i,j+1/2}\\ &-\big(B'^{+n}_{i,j-1/2}U^{+n}_{i,j-1/2}+B'^{-n}_{i,j-1/2}U^{-n}_{i,j-1/2}\big)\cdot S_{i,j-1/2}\Big]\end{aligned} \tag{15}$$

其中: $U^{+n}_{i+1/2,j}=U_{ij}^n+\Delta S_{ij}^{(\xi)}/(\Delta S_{ij}^{(\xi)}+\Delta S_{i-1,j}^{(\xi)})\cdot(U_{ij}^n-U_{i-1,j}^n)$

$$U^{-n}_{i+1/2,j} = U^n_{i+1,j} + \Delta S^{(\xi)}_{i+1,j} / (\Delta S^{(\xi)}_{i+1,j} + \Delta S^{(\xi)}_{i+2,j}) \cdot (U^n_{i+1,j} - U^n_{i+2,j})$$

$$U^{+n}_{i,j+1/2} = U^n_{ij} + \Delta S^{(\eta)}_{ij} / (\Delta S^{(\eta)}_{ij} + \Delta S^{(\eta)}_{i,j-1}) \cdot (U^n_{ij} - U^n_{i,j-1})$$

$$U^{-n}_{i,j+1/2} = U^n_{i,j+1} + \Delta S^{(\eta)}_{i,j+1} / (\Delta S^{(\eta)}_{i,j+1} + \Delta S^{(\eta)}_{i,j+2}) \cdot (U^n_{i,j+1} - U^n_{i,j+2})$$

求解隐式差分方程 (14) 式采用在每一个时间步上按拟流线坐标方向往返一次扫描的 Gauss-Seidel 线松驰迭代方法，即每一个时间步分别求解:

1. i 从大到小:

$$\hat{A}^n_{ij}\delta U^{\bar{n}}_{ij} + \hat{C}^n_{ij}\delta U^{\bar{n}}_{i,j-1} + \hat{B}^n_{ij}\delta U^{\bar{n}}_{i,j+1} = \Delta U^n_{ij} - \hat{E}^n_{ij}\delta U^{n-1}_{i-1,j} - \hat{D}^n_{ij}\delta U^{\bar{n}}_{i+1,j} \tag{16}$$

2. i 从小到大:

$$\hat{A}^n_{ij}\delta U^n_{ij} + \hat{C}^n_{ij}\delta U^n_{i,j-1} + \hat{B}^n_{ij}\delta U^n_{i,j+1} = \Delta U^n_{ij} - \hat{E}^n_{ij}\delta U_{i-1,j} - \hat{D}^n_{ij}\delta U^{\bar{n}}_{i+1,j} \tag{17}$$

(16) 和 (17) 均可用块追赶法求解，当扫描到出口边界和进口边界时，作进出口边界处理；叶栅壁面条件及周斯性条件则在构造系数矩阵 $\hat{A}_{ij}$、$\hat{B}_{ij}$ 和 $\hat{C}_{ij}$ 时予以考虑。

四、边界条件提法及数值处理

平面叶栅的求解域及各类边界如图 2 所示，由 Euler 方程的特征分析[4]，在进口边界 $\overline{AB}$ 上，一般有 $Ma_x < 1$，故应指定三个边界条件；在出口边界，一般仍有 $Ma_x < 1$，应给定一个边界条件；在叶栅壁面则给定法向速度为零这一滑移条件。

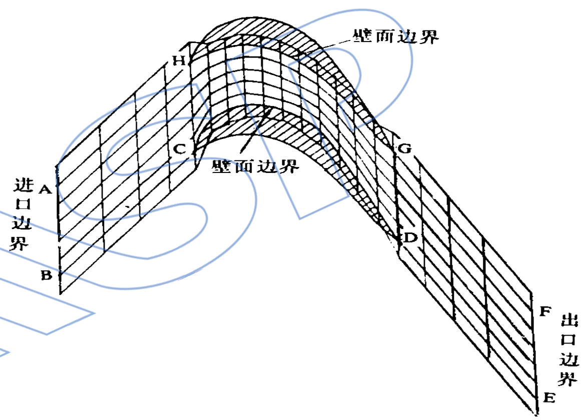

图 2　平面叶栅各类边界及网格划分示意图

以下逐个说明各种边界的数值处理。

1. 进口边界　给定总温 T_t、总压 P_t 和进气角 β_1，同时应用第二族波特征相容关系[4]:

$$\partial P/\partial t - \rho c\,\partial u/\partial t = -(u-c)(\partial P/\partial x - \rho c\,\partial u/\partial x) \tag{18}$$

或者

$$(\partial P/\partial u - \rho c)\,\partial u/\partial t = -(u-c)(\partial P/\partial x - \rho c\,\partial u/\partial x) \tag{19}$$

若记

$$\delta Z_{ij} = \Delta t \cdot \left(\frac{\partial Z}{\partial t}\right)_{ij} = \begin{cases} \overline{Z^{n+1}_{i,j}} - Z^n_{i,j} & (\text{第一次扫描时}) \\ Z^{n+1}_{i,j} - Z^n_{i,j} & (\text{第二次扫描时}) \end{cases} \tag{20}$$

则 (19) 式差分离散为

$$(dP/du - \rho c)^n_{2,j}\delta u_{1,j} = -[\lambda/(\lambda-1)][P^n_{2,j} - P^n_{1,j} - (\rho c)^n_{2,j}(u^n_{2,j} - u^n_{1,j})] - [\lambda/(\lambda-1)][\delta P_{2,j} - (\rho c)^n_{2,j}\delta u_{2,j}] \tag{21}$$

其中　$\lambda = (u-c)^n_{2,j}\Delta t/\Delta x_1$，　dP/du 利用等熵关系有:

$$dP/du = -P_t[2u\gamma/(\gamma+1)](1+\mathrm{tg}^2\beta_1)\{1-[(\gamma-1)/(\gamma+1)](1+\mathrm{tg}^2\beta_1)u^2\}^{1/\gamma-1} \tag{22}$$

因此由 (21)、(22) 可求出 $\delta u_{1,j}$，并 $u_{1,j}$ 的新值，再由给定的三个条件有:　$v = u\,\mathrm{tg}\beta_1$，

$T = T_t[1-(\gamma-1)/(\gamma+1)\cdot(u^2+v^2)]$, $p = p_t[(1-(\gamma-1)/(\gamma+1))\cdot(u^2+v^2)]^{\gamma/(\gamma-1)}$, $\rho = P\gamma/T$　则进口边界的所有流动参数及相应增量可全部算出。

2. 出口边界　出口边界给定出口压力 P_{exit}，另外应用两族流特征相容关系及第一族波特征相容关系，即

$$\partial v/\partial t = -u\partial v/\partial x,\quad \partial\rho/\partial t-(1/c^2)\partial P/\partial t = -u(\partial\rho/\partial x-(1/c^2)\partial P/\partial x)$$

$$\partial p/\partial t-\rho c\partial u/\partial t = -(u+c)(\partial P/\partial x+\rho c\partial u/\partial x)$$ 差分离散为：

$$\delta v_{IL,j} = -[\lambda_1/(1+\lambda_1)](v^n_{IL,j}-v^n_{IL-1,j}) + [\lambda_1/(1+\lambda_1)]\delta v_{IL-1,j} \equiv R_1 \tag{23}$$

$$(\delta\rho-\delta P/c^2)_{IL,j} = -[\lambda_1/(1+\lambda_1)][\rho^n_{IL,j}-\rho^n_{IL-1,j}-(1/c^2)(P^n_{IL,j}-P^n_{IL-1,j})] + [\lambda_1/(1+\lambda_1)][\delta\rho-(1/c^2)\delta P]_{IL-1,j} \equiv R_2 \tag{24}$$

$$(\delta P+\rho c\delta u)_{IL,j} = -[\lambda_2/(1+\lambda_2)][P^n_{IL,j}-P^n_{IL-1,j}+(\rho c)^n_{IL-1,j}(u^n_{IL,j}-u^n_{IL-1,j})] + [\lambda_2/(1+\lambda_2)](\delta P+\rho c\delta u)_{IL-1,j} \equiv R_3 \tag{25}$$

这里：$\lambda_1 = u^n_{IL-1,j}\Delta t/\Delta x_{IL}$，$\lambda_2 = (u+c)^n_{IL-1,j}\Delta t/\Delta x_{IL}$

最后可得：$\delta P_{IL,j} = 0$，$\delta\rho_{IL,j} = R_2$，$\delta u_{IL,j} = R_3/(\rho c)^n_{IL,j}$，$\delta v_{IL,j} = R_1$　于是出口边界的全部参数及增量可以算出。

3. 叶栅壁面边界　除给定法向速度为零外，补充法向动量方程，总焓不变及等熵假设，即给出如下关系　$V_n = 0$，$\partial P/\partial n = \rho V_\tau^2/R$，

$$\gamma p/[(\gamma-1)\cdot\rho] + 0.5(u^2+v^2) = h_o = \gamma P_t/[(\gamma-1)\rho_t],\quad P/\rho^\gamma = \text{const}$$

其中 V_τ 和 V_n 为壁面切向和法向速度，R 为壁面型线的曲率半径。由此可导出壁面边界上 U 的关系

$$\partial\rho/\partial n = \rho^2V_\tau^2/P\gamma R \tag{26}$$

$$S_y^{(\xi)}\partial\rho u/\partial n - S_x^{(\xi)}\partial\rho v/\partial n = (\rho^2V_\tau^2/P-\rho\gamma)\cdot V_\tau/\gamma R \tag{27}$$

$$S_x^{(\xi)}\rho u + S_y^{(\xi)}\rho v = 0 \tag{28}$$

$$\partial E/\partial n = (h_o/\gamma R)(\rho^2V_\tau^2/P) - \rho V_\tau^2/R \tag{29}$$

因壁面边界点不是计算点，关系式 (26)～(29) 的离散须作特殊处理。以下给出吸力面边界的数值处理，而压力面边界的处理则类似之。

从图 3 可以看出 (26)、(27)、(29) 的离散应针对 O、W、C 三点，其中 W 点为壁面点，C 点为辅助网格的计算点、O 点的值则应由计算点 A、B 的值插值决定：

即

$$U_o = \Delta n/\Delta x\cdot|S_x^{(\xi)}|\cdot U_A + (1-\Delta n/\Delta x)\cdot|S_x^{(\xi)}|U_B \equiv \alpha_W U_A + (1-\alpha_W)U_B$$

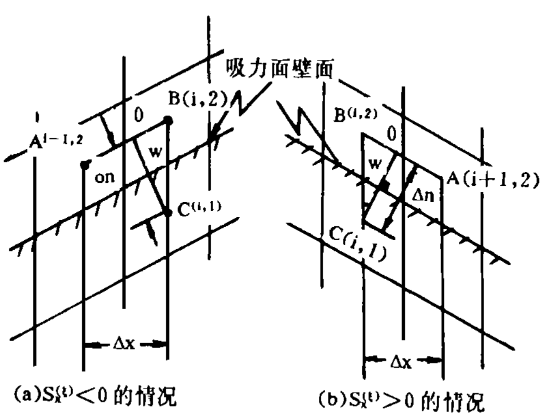

图 3　吸力面壁面表面的边界数值处理

由此可最终导出壁面边界附近 C、B、A 三个计算点增量间的隐式关系为：

$$\left(\hat{B}_{i,1},\ \hat{A}_{i,1}\right)\begin{pmatrix}\delta U_{i,2}\\ \delta U_{i,1}\end{pmatrix} = [RHS]$$

其中

$$\hat{B}_{i,1} = (1-\alpha_W)\begin{bmatrix} C_1^* - 1 & C_2^* & C_3^* & C_4^* \\ D_1^* & D_2^* - S_y^{(\xi)} & D_3^* + S_x^{(\xi)} & D_4^* \\ 0 & S_x^{(\xi)} & S_y^{(\xi)} & 0 \\ h_o C_1^* + \dfrac{\Delta n}{2R} V_\tau^2 & h_o C_2^* - \dfrac{\Delta n}{R} V_\tau S_y^{(\xi)} & h_o C_3^* + \dfrac{\Delta n}{R} V_\tau S_x^{(\xi)} & h_o C_4^* - 1 \end{bmatrix}$$

$$\hat{A}_{i,1} = \begin{bmatrix} C_1^* + 1 & C_2^* & C_3^* & C_4^* \\ D_1^* & D_2^* + S_y^{(\xi)} & D_3^* - S_x^{(\xi)} & D_4^* \\ 0 & S_x^{(\xi)} & S_y^{(\xi)} & 0 \\ h_o C_1^* + \dfrac{\Delta n}{2R} V_\tau^2 & h_o C_2^* - \dfrac{\Delta n}{R} V_\tau S_y^{(\xi)} & h_o C_3^* + \dfrac{\Delta n}{R} V_\tau S_x^{(\xi)} & h_o C_4^* + 1 \end{bmatrix}$$

$$[RHS] = \{\alpha_W/(1-\alpha_W)\}\hat{B}_{i,1}\cdot\delta U_A$$

A 点位置的确定，由图 3 可知：当 $S_x^{(\xi)} < 0$ 时，A 点取为 $(i-1,\ 2)$ 点；当 $S_x^{\xi} > 0$ 时，A 点取为 $(i+1,\ 2)$ 点。

$C_1^* = (\Delta n/2\gamma R)\{\rho^2 V_\tau^2(\gamma-1)/2P^2\}_W$

$C_2^* = (\Delta n/2\gamma R)\{\rho^2 V_\tau^2(\gamma-1)u/P^2 + 2\rho V_\tau S_y^{(\xi)}/P\}_W$

$C_3^* = (\Delta n/2\gamma R)\{\rho^2 V_\tau^2(\gamma-1)v/P^2 - 2\rho V_\tau S_x^{(\xi)}/P\}_W$

$C_4^* = = (\Delta n/2\gamma R)\{\rho^2 V_\tau^2(\gamma-1)/P^2\}_W$

$D_1^* = C_1^* V_\tau - (\Delta n/2\gamma R)(\rho V_\tau^3/P)_W$，$D_2^* = C_2^* V_\tau + (\Delta n/2\gamma R)[(\rho V_\tau^2/P-\gamma)\cdot S_y^{(\xi)}]_W$

$D_3^* = C_3^* V_\tau - (\Delta n/2\gamma R)[(\rho V_\tau^2/P-\gamma)\cdot S_x^{(\xi)}]_W$，$D_4^* = C_4^* V_\tau$

4. 周期性边界　周期性边界条件的处理运用带周期条件的块三对角矩阵的追赶法求解。

五、算 例 结 果

上述算法对航空部 624 所动中叶片进行无粘绕流计算，网格划分如图 2，沿拟流线方向共 41 个节点，其中叶片区 21 个节点，叶片前后延伸区各 10 个节点，节距线方向为 12 个节点，进口边界上给定：$P_t = 295685.6$ Pa，$T_t = 288$ K，$B_l = 50.4°$，出口边界上给定

$$P_{exit} = 102625.6 \text{ Pa}$$

收敛标准定为：

$$\underset{(i,j)}{MAX}\left| u_{i,j}^{n+1} - u_{i,j}^{n} \right| \le 0.0002 \qquad (5\text{-}1)$$

数值计算在 31 个时间推进步后达到 (5-1) 式的收敛标准。可见所需的时间步是相当少的。在 IBM 4341 计算机上 CPU 时间约 8 分钟。

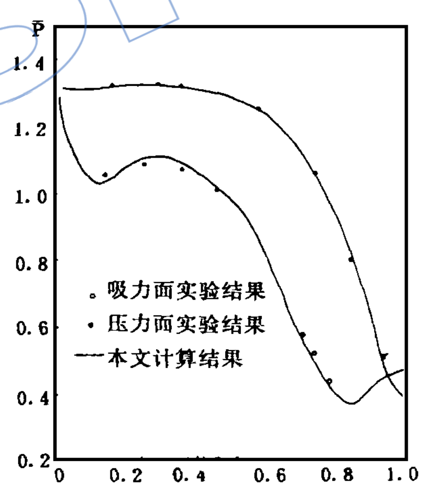

图 4　624 动中叶片壁面压力曲线

图 4 给出数值结果与实验的对比，可以看出，结果是令人满意的。

参 考 文 献

[1] Steger J. L.” Warming R. F., “Flux Vector Splitting of the Inviscid Gasdynamic Equations with Application to Finite-Difference Methods,” J. Computational Physics 40, 263-293 1981.

[2] MacCormack R. W., “Current Status of Numerical Solutions of the Navier-Stores Equations ”, AIAA-85-0032.

[3] 沈孟育、周盛。林保真，“叶轮机械中的跨音速流动”，科学出版社，1988。

COMPUTATION OF TRANSONIC FLOW IN A PLANE CASCADE WITH AN UNFACTORED FLUX SPLITTING IMPLICIT METHOD

Huang Dongtao, Shen Mengyu (*Qinghua University*)
Zhang Yaoke (*Computing Center, Academia Sinica*)

ABSTRACT MacCormack's flux splitting method has proven itself guite fast converging in numerical simulation of flows in converging-diverging nozzle. In this paper a development on the basis of this method is presented for Euler equations in order to obtain numerical solutions of inviscid transonic flow through a two-dimensional turbine cascade. In the present finite-area flux splitting implicit method the Gauss-Seidel line relaxation recommended by R. W. MacCormack is applied to solving the implicit descretization equations. But we abandon the predictor-corrector two-step method to cut down the computational time in every time-step, because it cannot enhance the accuracy of the steady soluton in flux-splitting methods. In our practical computation, in order to improve the accuracy of the solution, the right side of the difference equation is discretized to be of second order accuracy, the blade wall boundary conditions and the numerical results show that the present method retains the effective convergence rate (only about 30 timesteps to attain a steady solution) and yield numerical solutions conformable with the experimental results.

RELAXATION METHOD FOR TRANSONIC POTENTIAL FLOWS THROUGH 2-D CASCADE WITH LARGE CAMBER ANGLE

Wu Guohua and Peng Zeyan
(***Beijing University of Aeronautics and Astronautics***)

ABSTRACT A new trnsonic relaxation method is presented which can be used for computation of transonic, potential flows through two-dimensional cascade with large camber angle. A non-orthogonal mesh system composed of streamlines and straight lines parallel to y-axis is employed here. The governing equation expressed by the streamline coordinate system is solved in physical plane. Because of the streamline coordiate system, the governing equation is greatly simplified and the formulation of the finite difference scheme is also made correctly and easily. In the case of the cascade with large camber angle (especially for turbine) direct formulation of difference scheme from full-potential equation (instead of perturbation-potential equation) is suggested.

The numerical experimentations show that both the convergence and the stability of the finite difference scheme proposed here are satisfactory. The compuation results with acceptable accuracy can be obtained after 60 to 90 relaxation steps. The numerical examples also show that the results are in good agreement with experimental data and analytical solution (for Hobson-airfoil).

EXPERIMENTAL INVESTIGATION ON THE PERFORMANCE OF AN ANNULAR NOZZLE CASCADE of A HIGHLY-LOADED TRANSONIC TURBINE-STAGAE

Zhou Shiying, Li Jinnian, Huang Zhonghu (*Shenyang Aeroengine Research Institut*)
Yang Xuezhen and Zhang Tiansong (*Gas Turbine Establishment*)

ABSTRACT An annular nozzle cascade of a highly-loaded transonic turbine-stage has been tested in annular cascade wind tunnel, and its two plane cascades at the hub- and mid-sections—in a plane cascade wind tunnel. Their loss performance, exit flow angle, exit flow velocity as well as the flow velocity distribution on the blade-profile surfaces obtained in both cases are analyzed comparatively. It is proven that the experimantal results of the annular nozzle cascade are comprable with those of the plane cascades, particularlly, in quite good agreement at the mid-section. But at the hub-section a obvious difference between the experimental of both results occurs because of the local flow complexity in the annular nozzle cascade.