第 4 卷　第 4 期　　航空动力学报　　Vol. 4　No. 4
1989 年　10 月　　Journal of Aerospace Power　　Oct.　1989

一种求解涡轮平面叶栅的跨音松驰法

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一、引　　言

跨音松驰法是七十年代初发展的一种新方法。文献 [1] 是一篇关于跨音松驰法的奠基性文章，文章提出了求解小扰动势方程的混合差分格式。文献 [2] 发展了求解全位势方程的旋转混合差分格式。近年来，求解跨音全位势方程的方法有了更全面的发展[4,5]。作者于 80 年发展了一种求解大弯度二维叶栅的跨音松驰法[3]。本文将该法推广到求解涡轮平面叶栅跨音带激波流场，直接求解全位势方程，进一步提高边界区差分格式的精度，并注意初场的给法。

本法具有节约机时和内存的优点。数值算例表明，对于头部不大的涡轮平面叶栅，应用本方法可获得令人满意的结果，对于头部很大的涡轮平面叶栅，需对头部区域加密网格。

二、控 制 方 程

将直角坐标系下的全位势方程变换到 S-Y 座标系，则可获得全位势 Φ 的控制方程[6]：

$$(1-Ma^2)\Phi_{ss} = -\mathrm{tg}^2\beta\Phi_{ss} + 2\mathrm{tg}\beta\sec\beta\Phi_{ys} - \sec^2\beta\Phi_{yy} \tag{1}$$

式中 Φ_{ss}、β_{ys} 和 Φ_{yy} 分别为势函数 Φ 沿 S 向的二阶导数、沿 Y-S 向的混合导数和沿 Y 向的二阶导数；β 则为气流方向与 X 轴的夹角。为求解流场，还要用到能量方程和气体状态方程。

$$a_i^2 + (\gamma-1)V_i^2/2 = const \tag{2}$$

$$P = \rho RT \tag{3}$$

三、内点差分格式的建立

构造差分格式时，严格遵循讯号传播法则，同时还对迭代过程中的旧值和新值进行了适当的搭配[2]，以获得稳定性好和收敛速度快的差分格式。

1. 超音区：(1) 式的左边 Φ_{ss} 项取上风差分，其余导数均取中心差分。

$$\Phi_{ss左} = 2\frac{\Phi^{+}_{i,j}(\Delta s_{i-1,j}+\Delta s_{i-2,j}) - \Phi_{i,j}\Delta s_{i-1,j}}{s_{i-1,j}\Delta s_{i-2,j}(\Delta s_{i-1,j}+\Delta s_{i-2,j})}$$

$$+ 2\frac{\Phi_{i-2,j}\Delta s_{i-1,j} - \Phi^{+}_{i-1,j}(\Delta s_{i-1,j}+\Delta s_{i-2,j})}{\Delta s_{i-1,j}\Delta s_{i-2,j}(\Delta s_{i-1,j}+\Delta s_{i-2,j})} \tag{4}$$

$$\Phi_{ss右} = 2\frac{\Phi_{i+1,j}\Delta s_{i-1,j} - \Phi_{i,j}\Delta s_{i-1,j} - \Phi^{+}_{i,j}\Delta s_{i,j} + \Phi^{+}_{i-1,j}\Delta s_{i,j}}{\Delta s_{i-1,j}\Delta s_{i,j}(\Delta s_{i-1,j}+\Delta s_{i,j})} \tag{5}$$

$$\Phi_{yy} = 2\frac{\Phi_{i,j+1}\Delta y_{i,j-1} - \Phi^{+}_{i,j}(\Delta y_{i,j}+\Delta y_{i,j-1}) + \Phi^{+}_{i,j-1}\Delta y_{i,j}}{\Delta y_{i,j}\Delta y_{i,j-1}(\Delta y_{i,j}+\Delta y_{i,j-1})} \tag{6}$$

式中上标+ 表示新值。

$$\Phi_{ys}=\frac{(\Phi_y)_{i+1,j}\Delta s_{i-1,j}^2+(\Phi_y)_{i,j}(\Delta s_{i,j}^2-\Delta s_{i-1,j}^2)-(\Phi_y)_{i-1,j}\Delta s_{i,j}^2}{\Delta s_{i,j}\Delta s_{i-1,j}(\Delta s_{i-1,j}+\Delta s_{i,j})}\tag{7}$$

$$\Phi_y=\frac{\Phi_{i,j+1}\Delta y_{i,j-1}^2+\Phi_{i,j}(\Delta y_{i,j}^2-\Delta y_{i,j-1}^2)-\Phi_{i,j-1}\Delta y_{i,j}^2}{\Delta y_{i,j}\Delta y_{i,j-1}(\Delta y_{i,j}+\Delta y_{i,j-1})}\tag{8}$$

类似地可得 $(\Phi_y)_{i+1,j}$和 $(\Phi_y)_{i-1,j}$。

将 (4) ～ (8) 式代入到 (1) 式中，就可得到用于求解超音点流场的差分方程。

2. 亚音区:

(1) 式中的左端和右端的 Φ_{ss} 项均取中心差分，并引入松弛因子 ω，则有

$$\Phi_{ss}=2\frac{\Phi_{i-1,j}^{+}\Delta s_{i,j}-(1/\omega)(\Delta s_{i,j}+\Delta s_{i-1,j})\Phi_{i,j}^{+}}{\Delta s_{i,j}\Delta s_{i-1,j}+(\Delta s_{i,j}+\Delta s_{i-1,j})}$$

$$+2\frac{\Phi_{i+1,j}\Delta s_{i-1,j}-(1-1/\omega)(\Delta s_{i,j}+\Delta s_{i-1,j})\Phi_{i,j}}{\Delta s_{i,j}\Delta s_{i-1,j}(\Delta s_{i,j}+\Delta s_{i-1,j})}\tag{9}$$

Φ_{ys}和 Φ_{ss}的表达式分别与 (6) 式和 (7) 式相同。将 (6) ～ (9) 式代入 (1) 式中，则得到亚音区的差分格式。

可以看出，(4) 式为一阶精度，其余的导数表达式为二阶或接近二阶精度。格式稳定性和文献 [2] 类同。

四、边界点的差分格式

为进一步改善大弯度叶栅流场计算，本文采用精度高于文献 [3] 的物面边界点格式。

1. 槽道区 (即物面边界点):

$$(\Phi_{yy})_{i,1}=(\Phi_{yy})_{i,1+1/16}=2\frac{5\Delta y_{i,1}-16\Delta y_{i,2}}{\Delta y_{i,1}\Delta y_{i,1}(\Delta y_{i,1}+\Delta y_{i,2})}\Phi_{i,1}^{+}+2\frac{16\Delta y_{i,2}-9\Delta y_{i,1}}{\Delta y_{i,1}^2\Delta y_{i,2}}\Phi_{i,2}^{+}$$

$$+2\frac{9\Delta y_{i,1}-12\Delta y_{i,2}}{\Delta y_{i,1}\Delta y_{i,2}(\Delta y_{i,1}+\Delta y_{i,2})}\Phi_{i,3}^{+}-\frac{8}{\Delta y_{i,1}}(\Phi_y)_{i,1}\tag{10}$$

$$(\Phi_{yy})_{i,jN}=(\Phi_{yy})_{i,jN-1/16}=2\frac{5\Delta y_{i,jN-1}-16\Delta y_{i,jN-2}}{\Delta y_{i,jN-1}^2(\Delta y_{i,jN-2}+\Delta y_{i,jN-1})}\Phi_{i,jN}^{+}+2\frac{16\Delta y_{i,jN-2}-9\Delta y_{i,jN-1}}{\Delta y_{i,jN-1}^2\cdot\Delta y_{i,jN-2}}\Phi_{i,jN-1}^{+}$$

$$+2\frac{9\Delta y_{i,jN-1}-12\Delta y_{i,jN-2}}{\Delta y_{i,jN-1}\Delta y_{i,jN-2}(\Delta y_{i,jN-1}+\Delta y_{i,jN-2})}\Phi_{i,jN-2}^{+}+\frac{8}{\Delta y_{i,jN-1}}(\Phi_y)_{i,jN}\tag{11}$$

2. 进、出口区:

$$(\Phi_{yy})_{i,1}=\frac{2}{\Delta y_{i,1}}\left[\frac{\Phi_{i,2}^{+}-\Phi_{i,1}^{+}}{\Delta y_{i,1}}-(\Phi_y)_{i,1}\right]\tag{10'}$$

$$(\Phi_{yy})_{i,jN}=\frac{2}{\Delta y_{i,jN-1}}\left[(\Phi_y)_{i,jN}-\frac{\Phi_{i,jN}^{+}-\Phi_{i,jN-1}^{+}}{\Delta y_{i,jN-1}}\right]\tag{11'}$$

五、线松驰迭代方程的建立

1. 计算域: 如图 1 所示。在槽道区采用等距网格，在进、出口区采用逐次递增网格，即 x 向步长为 Δx，$2\Delta x$，……。

2. 边界条件　在进口边界 H 截面上，给定气流的马赫数 Ma_1、气流角 β_1 及气体压强 P_1;

在出口截面 DE 上给定气体压强 P_2；在尾缘处满足广义库塔条件；在进、出口区周期边界上满足周期性条件；在物面上满足相切条件。

3. 线松驰迭代的差分方程：将 (4) ～ (11) 式分别代入 (1) 式，即可得差分方程，其一般形式在迭代时，以 i 为常数的线作为松驰线，由左向右逐线松驰迭代求解。

$$A_{i,j}\Phi^{+}_{i,j-1} + B_{i,j}\Phi^{+}_{i,j} + C_{i,j}\Phi^{+}_{i,j+1} = D_{i,j} \tag{12}$$

对于槽道区：$j = 1, 2\cdots, jN$；$i = ie, ie+1, \cdots, it$；

对于进、出口区：$j = 1, 2\cdots, jN+1$；$i = 1, 2, \cdots, ie-1$ 或 $i = it+1, it+2, \cdots, im$。

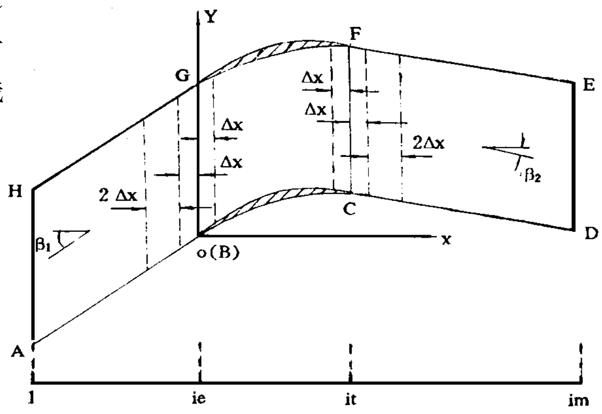

图 1　计算域示意图

将 (12) 式写成矩阵的形式：

$$\begin{bmatrix} {}_1 & o & o & \cdots & o & o \\ {}_2 & C_{i,2} & o & \cdots & o & o \\ {}_3 & B_{i,3} & C_{i,3} & \cdots & o & o \\ \vdots & \vdots & & & \vdots & \vdots \\ & \cdots & o & \cdots & A_{i,jN} & B_{i,jN} \end{bmatrix} \begin{bmatrix} \Phi_{i,1} \\ \Phi_{i,2} \\ \vdots \\ \vdots \\ \Phi_{i,jN} \end{bmatrix} = \begin{bmatrix} D_{i,1} \\ D_{i,2} \\ \vdots \\ \vdots \\ D_{i,jN} \end{bmatrix} \tag{13}$$

$$\begin{pmatrix} B_{i,1} & C_{i,1} & o & o & \cdots & o & o \\ A_{i,2} & B_{i,2} & C_{i,2} & o & \cdots & o & o \\ o & A_{i,3} & B_{i,3} & C_{i3} & \cdots & o & o \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & & A_{i,jN} & B_{i,jN} & C_{i,jN} \\ -1 & \cdots & \cdots & \cdots & \cdots & 1 & o \end{pmatrix} \begin{bmatrix} \Phi_{i,1} \\ \Phi_{i,2} \\ \vdots \\ \vdots \\ \vdots \\ \Phi_{i,jN} \\ (\Phi)_{i,1} \end{bmatrix} = \begin{bmatrix} D_{i,1} \\ D_{i,2} \\ \vdots \\ \vdots \\ \vdots \\ D_{i,jN} \\ D_{i,jN+1} \end{bmatrix} \tag{14}$$

(13) 式为槽道区方程组，它是标准三对角矩阵，可用追赶法求解。(14) 式为进、出口区方程组，它基本上是三对角矩阵，作一定变换后仍可用追赶法求解。

六、稳定性条件、初场选取和流线迭代

由文献 [2]，差分格式在超音区是稳定的。对于亚音区，由伪时间相关分析按二阶线性偏微分方程的特征理论，得本文差分格式的稳定条件是[3]

$$\omega < 1 + \Delta s_{i-1,j} / \Delta s_{i,j}$$

跨音流场的特征是强烈的非线性，而特大弯度的涡轮叶栅使这一非线性特性变得更加突出。具体体现为对初场的敏感性，即对初场要求更高。可以证明，对于线性问题，若差分格式稳定，则收敛解和初场无关。然而，对于非线性问题，有时因初场给得不好而不能获得收敛解。作者的数值经验表明，要根据通道的几何性质与物理流动性质相的一致原则来构造初场。例如，收 $Hobson$ 叶型的数值计算采用如下形式定义 Φ 的初场。

$$\Phi_{i,j} = \begin{cases} q_1(x_{i,j}\cos\beta_1 + y_{i,j}\sin\beta_1) & i < ie \\ q_1\cos\beta_{i,jN-j+1}/\cos\beta_1\Delta s_{i-1,j} + \Phi_{i-1,j} & i \geq ie \end{cases}$$

还有其它形式的初场给法[7]。还应指出，构造的初场 Φ值应该是基本光滑的。

由于本文求解的遵循方程是沿流线导出的，流线还是构成求解流场时网格的基本部分，因此本计算方法求解过程始终贯穿流线迭代。由等分槽道线组成的初始流线开始，便可进行跨音松驰迭代计算，到一定精度后，就可进行流线修正。然后对修正后的新流线再进行松驰迭代计算，直到各站流管相对误差和流场中各点马赫数达到给定精度后，则迭代结束。

七、数 值 结 果

图 2 所示为 Hobson 叶栅的算例。可以看出，用本文的方法获得的数值解和 Hobson 叶型的解析解很相近。它表明可用本方法计算大弯度跨音涡轮叶栅。

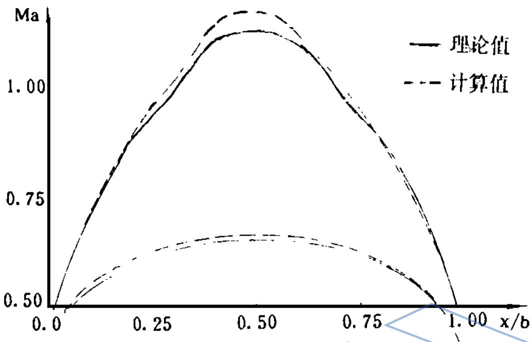

图 2 Hobson 叶栅算例

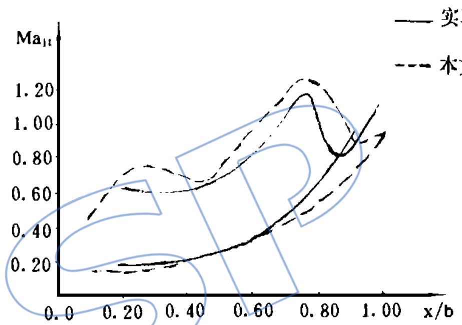

图 3 624 叶栅算例

图 3 所示，叶盆 Ma 数分布和实验值相近，叶背相差较多。这是由于本叶栅属于大头部前缘所造成。为改进计算结果，需采取头部前缘特殊处理或加密网格。

八、结 束 语

1. 本方法可以推广到 ADI 方法。

2. 本方法应用前提为“无旋流动”，因此流场中激波前的法向马赫数以不大于 1.3 为宜，否则会导致误差增大。

3. 本方法具有计算速度快、要求计算机内存小的优点。

4. 本方法可用于计算带激波的跨音速涡轮平面叶栅。

5. 为改进本方法，对于大头部前缘的涡轮叶栅的前缘处理，尚待改进。

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COMPUTATION OF TRANSONIC FLOW IN A PLANE CASCADE WITH AN UNFACTORED FLUX SPLITTING IMPLICIT METHOD

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ABSTRACT MacCormack's flux splitting method has proven itself guite fast converging in numerical simulation of flows in converging-diverging nozzle. In this paper a development on the basis of this method is presented for Euler equations in order to obtain numerical solutions of inviscid transonic flow through a two-dimensional turbine cascade. In the present finite-area flux splitting implicit method the Gauss-Seidel line relaxation recommended by R. W. MacCormack is applied to solving the implicit descretization equations. But we abandon the predictor-corrector two-step method to cut down the computational time in every time-step, because it cannot enhance the accuracy of the steady soluton in flux-splitting methods. In our practical computation, in order to improve the accuracy of the solution, the right side of the difference equation is discretized to be of second order accuracy, the blade wall boundary conditions and the numerical results show that the present method retains the effective convergence rate (only about 30 timesteps to attain a steady solution) and yield numerical solutions conformable with the experimental results.

RELAXATION METHOD FOR TRANSONIC POTENTIAL FLOWS THROUGH 2-D CASCADE WITH LARGE CAMBER ANGLE

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ABSTRACT A new trnsonic relaxation method is presented which can be used for computation of transonic, potential flows through two-dimensional cascade with large camber angle. A non-orthogonal mesh system composed of streamlines and straight lines parallel to y-axis is employed here. The governing equation expressed by the streamline coordinate system is solved in physical plane. Because of the streamline coordiate system, the governing equation is greatly simplified and the formulation of the finite difference scheme is also made correctly and easily. In the case of the cascade with large camber angle (especially for turbine) direct formulation of difference scheme from full-potential equation (instead of perturbation-potential equation) is suggested.

The numerical experimentations show that both the convergence and the stability of the finite difference scheme proposed here are satisfactory. The compuation results with acceptable accuracy can be obtained after 60 to 90 relaxation steps. The numerical examples also show that the results are in good agreement with experimental data and analytical solution (for Hobson-airfoil).

EXPERIMENTAL INVESTIGATION ON THE PERFORMANCE OF AN ANNULAR NOZZLE CASCADE of A HIGHLY-LOADED TRANSONIC TURBINE-STAGAE

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ABSTRACT An annular nozzle cascade of a highly-loaded transonic turbine-stage has been tested in annular cascade wind tunnel, and its two plane cascades at the hub- and mid-sections—in a plane cascade wind tunnel. Their loss performance, exit flow angle, exit flow velocity as well as the flow velocity distribution on the blade-profile surfaces obtained in both cases are analyzed comparatively. It is proven that the experimantal results of the annular nozzle cascade are comprable with those of the plane cascades, particularlly, in quite good agreement at the mid-section. But at the hub-section a obvious difference between the experimental of both results occurs because of the local flow complexity in the annular nozzle cascade.