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计算轮盘破裂转速的大变形解析法

北京航空航天大学　洪其麟　王　屏

【摘要】 本文为轮盘破裂转速研究课题组的研究报告之一。文中提出的大变形解析法是计算轮盘破裂转速的一种新方法，首先推导了基本方程，其次叙述了计算步骤，并对几个试验盘作了数值计算。最后将计算结果与试验结果作了对比，误差均在 4% 以内。作者认为，此法简便易行，准确度符合工程需要，适应范围较大，可以在发动机轮盘研制工作中应用。

一、前　言

轮盘是发动机的关键件之一，由于发动机在加速过程中转速瞬间超过、燃油调节器失灵或加力燃烧室故障，均可能导致轮盘超转。特别在控制系统失灵或轴损坏脱开时，更可能使轮盘超转甚至破裂。据统计，所有轮盘和绝大多数轮缘破裂均属于非包容性故障，轮盘的破裂碎片打穿机匣后，便可能切断油路或操纵系统，穿过油箱或座舱，对乘员和飞机造成严重危害。为了保证安全，一般建议在飞行包线内的任何情况下，轮盘最低的破裂转速不得小于最大允许的稳态转速的 122%[1]。因此，对轮盘破裂转速的准确预测和试验就是相当重要了。由于轮盘在工作时承受着相当大的载荷，特别在转速接近破裂转速时，所产生的变形已达到材料硬化区的大变形。所以需要采用大变形的概念，即几何非线性和材料非线性概念。在这样的双重非 线性问题中，精确求解破裂转速是相当困难的。本文采用一种简化解析法即大变形解析法，来求解硬化材料轴对称平面应力轮盘的破裂转速。

二、大变形解析法的基本方程

1. 假设条件　(1) 符合简单加载条件。在塑性变形时，物体体积不变；等效应力与应变间成幂函数关系；特别是外载按比例单调增加；(2) 轮盘处于平面应力状态；(3) 采用大变形概念。设 L_i 是沿任一主轴方向的微小线元素的瞬时长度，其原长为 $(L_i)_o$，则其应变 ϵ_i 为 $\epsilon_i = \ln [L_i/(L_i)_o]$，或 $e^{\epsilon_i} = L_i/(L_i)_o$；(4) 弹性变形比塑性变形小得多，可以略去不计。

2. 等厚盘的基本方程　对旋转的轴对称平面应力状态的轮盘，可以建立以下九个方程以解出 σ_r、σ_θ、ϵ_r、ϵ_θ、ϵ_z、γ、τ、h 和 u 等九个未知量。γ 和 τ 分别表示八面体剪应变和剪应力，h 为轮盘微元瞬时厚度，u 为径向位移。平衡方程[3]：考虑一外径 r_k、厚度为 h_o 的旋转等厚盘，如图 1 所示。轮盘材料密度为 ρ，旋转角速度为 ω，其平衡方程为：

$$(r+u)\,[d(\sigma_r h)/d(r+u)] = (\sigma_\theta - \sigma_r)h - \rho\omega^2 r^2 h_o\,[(r+u)/r]\,[dr/d(r+u)] \quad (1)$$

几何方程：采用大变形概念后，得：

$$e^{\epsilon_r} = d(r+u)/dr \quad , \quad e^{\epsilon_\theta} = (r+u)/r \quad , \quad e^{\epsilon_z} = h/h_o \quad (2)$$

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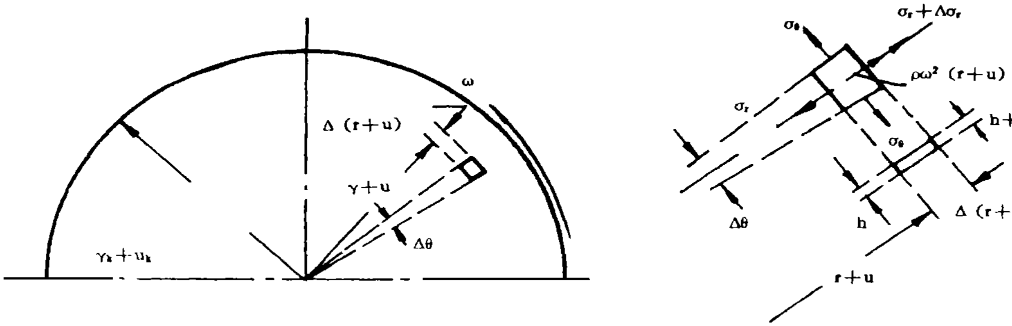

图 1 轮盘微无体受力平衡图

本构方程：材料进入塑性变形后，得：

$$\epsilon_r = (\gamma/3\tau)(\sigma_r - \sigma_\theta/2) + \alpha T, \quad \epsilon_\theta = (\gamma/3\tau)(\sigma_\theta - \sigma_r/2) + \alpha T \qquad (3)$$

式中的 α 为材料线膨胀系数，T 为温度差。以及

$$\epsilon_r + \epsilon_\theta + \epsilon_z = 0 \qquad (4)$$

$$\tau = (\sqrt{2}/3)(\sigma_r^2 - \sigma_r\sigma_\theta + \sigma_\theta^2)^{1/2} \qquad (5)$$

$$\gamma = \gamma(\tau) \qquad (6)$$

如果消去厚度 h 和位移 u，则可得以下七个方程：

$$r(d\sigma_r/dr) + r\sigma_r(d\epsilon_z/dr) = (\sigma_\theta - \sigma_r)e^{(\epsilon_r - \epsilon_\theta)} - \rho w^2 r^2 e^{(-\epsilon_z)} \qquad (7)$$

$$r(d\epsilon_\theta/dr) = e^{(\epsilon_r - \epsilon_\theta)} - 1 \qquad (8)$$

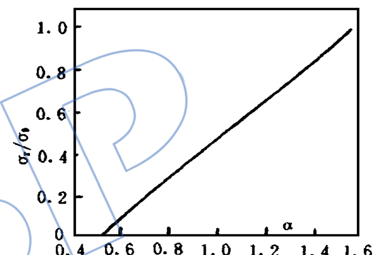

图 2 主应力比与参变数 α的关系图

以及 (3)、(4)、(5) 和 (6) 式。

如果将 ϵ_z 用 ϵ_r 和 ϵ_θ 表示，从 σ_r、σ_θ、ϵ_r 和 ϵ_θ 中消去两个，对每一种材料的 τ 和 γ 有一定的函数关系，且可由试验获得。则未知量减少到三个，可用 (6)、(7) 和 (8) 三式求解 σ_r、σ_θ 和 τ，但仍须用逐次逼近法。为避免这些困难，引入参变数 α。由于为平面应力问题，$\sigma_z = 0$，故在 $\sigma_r - \sigma_\theta$ 平面上，屈服轨迹为一椭圆。其长轴为 3τ，短轴为 $\sqrt{3}\tau$，并分别与 σ_r 和 σ_θ 成 45°。于是得出并解得：

$$\sigma_r = \sqrt{6}\tau\sin(\alpha - \pi/6), \quad \sigma_\theta = \sqrt{6}\tau\sin(\alpha + \pi/6) \qquad (9)$$

σ_r/σ_θ 随 α 的变化规律如图 2 所示，从图中可见，σ_r/σ_θ 与 α 基本上成线性变化。

将 (9) 和 (3) 式代入 (7) 和 (8) 式，并求解得：

$$r(d\alpha/dr) = (CE - FB)/(AE - DB), \quad r(d\gamma/dr) = (FA - CD)/(AE - DB) \qquad (10)$$

式中

$$A = 2\sin(\alpha + \pi/3) - 2\sin(\alpha - \pi/6)(\gamma\cos\alpha/\sqrt{2})$$

$$B = 2\sin(\alpha - \pi/6)\left[(d\tau/d\gamma)/\tau - \sin\alpha/\sqrt{2}\right]$$

$$C = 2\cos\alpha e^{-\gamma\sqrt{3/2}\cos\alpha} - \sqrt{2}\rho\omega^2 r^2 e^{(\gamma\sin\alpha/\sqrt{2} + 2\alpha_t T)}/(\sqrt{3}\tau) + 4r\sin(\alpha - \pi/6)(d\alpha T/dr)$$

$$D = 2\sin(\alpha - \pi/6)\gamma, \ E = -2\sin(\alpha + \pi/3), \ F = 2\sqrt{2}\left[1 + r(d\alpha T/dr) - e^{-\gamma\sqrt{3/2}\cos\alpha}\right]$$

根据 (10) 式可以解出 α 和 γ。

3. 非等厚盘的简化方法[3] 在工程计算上，常用一系列等厚圆环代替非等厚圆盘。设 j

环外半径处的参量为 α_j 和 γ_j，$j+1$ 环内半径处的参量则为 α'_{j+1} 和 γ'_{j+1}。根据联接面作用力相等条件得：

$$\tau'_{j+1} = \tau_j M \sin(\alpha_j - \pi/6)/\sin(\alpha'_{j+1} - \pi/6) \quad (11)$$

式中，$M = h_j/h_{j+1}$，h_j 和 h_{j+1} 分别为 j 和 $j+1$ 环的厚度。根据联接面的径向位移相等条件得：

$$\gamma'_{j+1} = \gamma_j \sin(\alpha_j + \pi/3)/\sin(\alpha'_{j+1} + \pi/3) \quad (12)$$

通过简单拉伸试验得到 $\tau-\gamma$ 关系后，即可按 (11) 和 (12) 式根据 α_j 和 γ_j 解出 α'_{j+1} 和 γ'_{j+1}。

4. 边界条件　对实心轮盘，设 r_k 为轮缘半径，则因 $r/r_k = 0$ 时，$\sigma_r = \sigma_\theta$，故 $\alpha = \pi/2$。对空心轮盘，设 r_o 为中心孔半径，因 $r/r_k = r_o/r_k$ 时，$\sigma_r = 0$，故 $\alpha = \pi/6$。在 $r/r_k = 1$ 时，σ_r/σ_{rk}。σ_{rk} 为轮缘处径向应力，可用下式计算：

$$\sigma_{rk} = F/[2\pi(r_k + u_k)h_k] \quad (13)$$

式中，F 为叶片和轮缘塔形块的离心力，其数值可按下式计算。

$$F = [m(r_k + u_k + \Delta) + w(r_k + u_k + l/2)]\omega^2 \quad (14)$$

式中，m 和 w 分别为轮缘塔形块和所有叶片的总质量，Δ 为塔形块质心到外缘距离，$l/2$ 为叶片质心到外缘距离。u_k 为外缘径向位移。

$$u_k = r_k[e^{\gamma_k \sin(\alpha_k + \pi/3)/\sqrt{2} + (\alpha_t T)_{r_k}} - 1], \quad h_k = h_o e^{-\gamma_k \sin\alpha_k/\sqrt{2} - 2(\alpha_t T)_{r_k}} \quad (15)$$

三、计算步骤及验证

当轮盘的尺寸形状、材料性能和工作环境等参数确定后，不同的转速有不同的应力场和应变场。如在某一转速下旋转，轮盘内有一点的八面体剪应变 γ 达到其极限 γ_b 时，则认为此盘破裂，该转速则为破裂转速。γ_b 为单向拉伸试验中名义应力达到材料强度极限 σ_b 值的八面体剪应变值，如 γ_b 为已知，则可按下列步骤算出轮盘的破裂转速值。

1. 由平均应力法算出破裂转速值，并将其作为本法的初始值 ω_i。

2. 将轮盘分为 k 个等厚圆环，根据边界条件确定中心孔处的 $\alpha'_1 = \alpha = \pi/6$，并设 $\gamma'_1 = \gamma_b$。然后从内往外逐环计算。

3. 根据内环内半径处的 α' 和 γ'_j 值算出 (10) 式中各系数值，并用 (10) 式算得内环外半径处的 α 和 γ_j 值。

4. 由两等厚圆环间的联接条件 (11) 和 (12) 式，求得外环内半径处的 α'_{+1} 和 γ'_{j+1} 值。

5. 重复第 3 和 4 步，直到求得最外一个环即 k 环外半径处的 α_k、γ_k、σ_{rk}。将 α_k 和 γ_k 代入 (15) 式，可以求出 u_k 和 h_k 值。再将此二值代入 (13) 式，便可求得 F 值。

6. 由 F 值通过 (14) 式以求得 ω_{i+1} 值。如果 $|\omega_{i+1} - \omega_i| \le \delta$ (δ 为给定的某一小值)，则停止迭代，此时 ω_{i+1} 即为破裂角速度 ω_b。否则将 ω_{i+1} 作为 ω_i，再重复 2– 6 步。

在前面推导公式中，曾假设为简

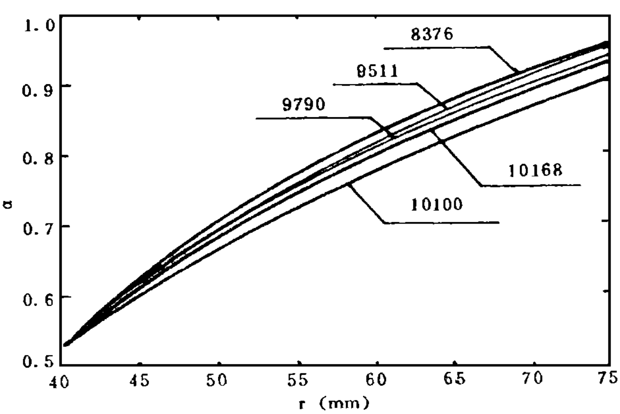

图 3　参变数 α 随转速 n 的变化图

单加载条件。众所周知，轮盘旋转时所受外载是按比例增加的，且主应力方向不变。如果某点主应力之比随转速的变化而不变，则符合简单加载条件。主应比 σ_r/σ_θ 取决于参变量 α，如果 α 值随转速的变化量很小，则基本符合简单加载条件，说明以上计算结果可信。现将一个有温度场、有轮缘外载的变厚盘的 α 值随转速的变化关系示于图 3 中，从图上可以看出，随着转速变化，α 值变化非常小，因此可以认为基本符合简单加载条件。

四、破裂转速的计算值与试验值比较

下面将四个试验盘的破裂转速试验值和用平均应力法、小变形解析法和大变形解析法的计算值列在表 1[4] 中进行对比，并得出该三种计算方法的相对误差。破裂转速单位为 rpm。

表 1 破裂转速的计算值与试验值对比表

盘号	试验盘特点	破裂转速试验值	平均应力法		小变形解析法		大变形解析法	
			计算值	误差%	计算值	误差%	计算值	误差%
1	有机玻璃等厚盘	9552	12050	26.15	9460	0.96	9449	1.08
2	A3 钢等厚盘	11278	10720	4.95	10694	5.18	10849	3.8
3	A3 钢变厚正轮毂盘	11850	11356	4.17	11243	5.12	11426	3.58
4	A3 钢变厚偏轮毂盘	11430	11356	0.65	11243	1.64	11426	0.035

五、结 论

比较表 1 中的破裂转速试验值和计算值，可得出以下结论:

1. 用平均应力法计算轮盘破裂转速只适合于韧性材料盘，误差一般在 5% 以内[4]。该法简便易算，在方案设计时，可用来初估轮盘破裂转速值。

2. 小变形解析法只适用于计算脆性材料轮盘的破裂转速值，对其它材料均不宜使用。

3. 用大变形解析法计算轮盘破裂转速，对各种情况均较合适，误差一般在 4% 之内，此法可以在轮盘详细设计中使用。

4. 对于偏轮毂盘，如其偏位值不很大，与同样材料和形状的正轮毂盘比较，其破裂转速试验值约减少 3.5%。因此，也可以使用大变形解析法。

5. 对轮盘作高温破裂试验时，因所用加热器功率不够，致使轮盘在高速旋转后温度急剧降低。为了使轮盘能在规定温度场下破裂，故加温后，急剧增加转速，直到轮盘破裂。由于加载速率过大，使轮盘破裂转速的试验值较大，从而与计算值相比偏差较大。例如 A3 钢等厚盘作高温破裂试验时，轮缘温度为 515°C，中心孔边温度为 400°C，其破裂转速的试验值为 9750rpm，而其计算值为 8525rpm，误差为 12.5%。如按正常加速轮盘，其试验值应更接近计算值，因此在此条件下也可以使用大变形解析法。

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method is proposed for yield surface correction. The other is how to deal with $\Delta\alpha_z$ and $\Delta\epsilon_z$ of plane stress problem becauase they are not equal to zero. The solution of this problem is also provided in the program. Several numerical examples are given to check the reliability of the MTEPCA program. The stress-strain hysteresis loops of the point A (danger point) on the analogue sample under cyclic loading are analysed for the structural integrity research.

VARIABLE AMPLITUDE LCF LIFE PREDICTION METHODS

Wang Lüsheng, Ru Meixiu, Cao Fenglan, Gong Mengxian
(***Gas Turbine Establishment***)

ABSTRACT This paper describes our variable amplitude LCF life prediction system based on the strain fatigue theory. The local stress-strain method modified by us is more accurate than the conventional local stress-strain method, This method is provided for disk life design. The presented system is also capable of accurate prediction of the variable amplitude LCF life according to the statistic data of smooth round bar specimen LCF lives to failure, so it is quite a convenient and simple experimental estimation method. Finally, the new method used in the presented system for detemining the reliability life of rotating disks is better than the method in EGD-3 stress standard.

STRESS INTENSITY FACTORS AND THE PROPAGATION OF SURFACE CRACK IN TURBINE DISK

Zheng Guanghua and Xia Deyi
(***Beijing University of Aeronautics and Astronautics***)

ABSTRACT A displacement method is provided to determine the stress intensity factors of nodes on crack surface in cracked body. The K_I at the points along crack front of surface crack are obtained from the curve of K_I versus r by the extrapolation. The K_I of semi-circular and semi-elliptical surface crack in the finite-thickness plates subjected to tensile load are calculated by the three-dimensional finite-element method. The photoelastic experiments with stress frozen method are made to examine the accuracy of the calculated results by the present method. Based on the expression proposed by Elber and Newman for the crack growth rate, the crack growth pattern and the life prediction of semi-elliptical crack in a turbine disk of a jet engine are investigated. It is shown that the method presented in this paper is simple and convenient to analyse surface crack problems of engineering components.

A LARGE ANALYTICAL DEFORMATION METHOD FOR PREDICTION OF DISK BURST SPEED

Hong Qilin and Wang Ping
(***Beijing University of Aeronautics and Astronautics***)

ABSTRACT A new method has been elaborated to predict the burst speed of aeroengine disks. The disks should overspeed or even break due to malfunction of control system, shaft decoupling or afterburning failure. In order to avoid catastrophic failure of an engine, it is required to allow a margin of the disk burst speed. When the maximum rotor speed approaches to the disk burst speed, the plastic strain attains to large value in strain-hardening range. So the concepts of material and geometrical nonlinearities are employed, but it is very difficult to obtain the solution of the double-nonlinear problems. An approximate large deformation large analytical method is proposed firstly. The basic equations are derived according to the variable thickness and temperature of the disks. Secondly, the computational procedure is described and the burst speeds of four tested disks are obtained from the calculations. At last, the calculated results are compared with the test data, and the differences are less than four percent. Therefrom, it is proved that the new method is simple, convinent and accurate for engineering application as well as it is applicable to a wide variety of disks.

APPLICATION OF PHOTOELASTIC COATING AND PHOTOPLASTICITY TO ELASTOPLASTIC STRESS ANALYSIS OF A ROTATING DISK

Dong Benhan and Gao Pengfei

(***Shenyang Aeroengine Research Institute***)

ABSTRACT A experimental method is developed for elastoplastic stress analysis of a rotating disk by means of photoelastic coating and photoplasticity. The experimental results are obtained from elastoplastic strain measurements of a model disk and a compressor disk. The isochromatic fringe fields of rotating disks are recorded by two systems which are designed by authers and composed of derotator、strobe light、photoelectric detector and camera. Strips method and difference method are proposed to separate strains in the photoelastic coating. The elastoplastic stresses of the rotating disk are calculated from experimental data according to small deformation elastoplastic theory. This experimental method has the advantage over other methods in noncontact measurement of the whole field.

A THREE-DIMENSIONAL ELASTIC STRESS ANALYSIS FOR AIR-COOLED TURBINE BLADES OF ORTHOTROPIC MATERIAL WITH CONTOURED PASSAGE

Xu Heshan, Wang Zhengtao, Hao Yanhua

(***Shenyang Aeroengine Research Institute***)

ABSTRACT A three-dimensional elastic stress analysis method is provided for air-cooled turbine blades of orthotropic material with contoured passages. The analytical results are presented for a real air-cooled turbine blade. The sectional linear and nonlinear assumptions are adopted to divide a blade into elements and to form the geometrical and mechanical models, the coordinate transformation is employed to set up an elastic matrix of the elements, and the sectional linear interpolation is used to consider the effect of the temperature on the blade metal, so as to calculate the directionally-solidified air-cooled turbine blades with complicated contoured passages and the high temperature differences between the inner and outer walls. The method and the program have been verified and the accuracy could meet the requirements of engineering.