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# 随机有限元分析定向结晶涡轮叶片在随机压力场中的可靠性

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本文采用随机有限元法和二阶矩 Hasofer-Lind 方法, 分析随机压力场对于定向结晶叶片可靠性的影响。利用有解析表达式的 $\rho$、$\omega$做为随机量采用 Monte-Carlo 方法计算可靠性指标, 与本文编制的随机有限元程序计算的可靠性指标对比, 以验证后者的正确性。

1. 随机压力场计算模型及节点等效力　将叶片受到的压力场, 按照三维有限元所分网格形成许多小块 (图 1), 并认为在每一块上作用有垂直于表面的均匀压力, 当单元划分很细时, 可保持真实压力场的原貌。假定压力场是随机正态分布 (如非正态分布时可转化为正态分布), 并通过 $p_1$、$p_2 \cdots p_n$ 的均值和方差来反映, 同时假定其随机变化都是相互独立的。

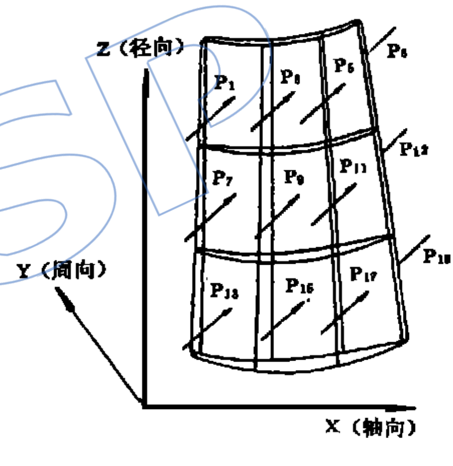


图 1　计算模型网格

当气体压力垂直于叶片表面且其载荷密度为 $q_o$, 则单元内第 $i$ 个节点等效载荷列阵为:

$$\{F_i\} = \int_{-1}^{1}\int_{-1}^{1} N_i q_o \{R\} d\xi d\zeta \qquad (1)$$

$\{R\}$ 为座标转换列阵。由于 $q_o$ 作用在叶片的叶盆、叶背的座标方向不同, 而基本随机变量 $X_1$、$X_2 \cdots X_n$ 均取压力为正, 吸力为负, 这样 $q_o$ 与基本随机变量的关系需考虑座标 $\eta$的正负 (见图 2), 可写为:

$$q_o = -\eta X_k \qquad (2)$$

由于 $\eta = \pm 1$ 为受压的边界面 (叶盆表面 $\eta = +1$, 叶背表面 $\eta = -1$)。将 (2) 式代入 (1) 式, 可写成:

$$\{F_i\} = \int_{-1}^{1}\int_{-1}^{1} N_i \{R\} d\xi d\zeta (-\eta X_k) \qquad (3)$$

$$\left\{\frac{\partial F_i}{\partial X_k}\right\} = \int_{-1}^{1}\int_{-1}^{1} N_i \{R\} d\xi d\zeta (-\eta) \qquad (4)$$

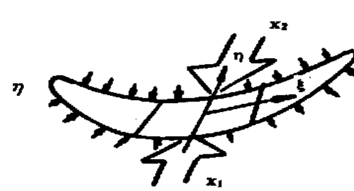


图 2　叶型相对座标示意图

将各单元 $\{F_i\}$ 叠加, 可得到压力场引起的载荷列阵 $\{F\}_p$。如还要考虑其它载荷 (如体积离心力等等), 可得总载荷列阵 $\{p\}$。同样 $\partial\{F_i\}/\partial X_k$ 也按上述方法叠加。

2. 节点位移导数及应力导数的求法　在有限元素法中结构的刚度方程为:

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$$[K]\{U\} = \{P\}$$

$[K]$ 为三维 20 节点单元、正交异性材料的刚度矩阵, 引入相应的位移边界条件, 可求出节点位移 $\{U\}$。右端项 $\{P\}$ 为载荷列阵。因压力随机变量与 $[K]$ 中的物理量无关, (暂不考虑材料物理量的随机变化), 故上式求导时可写成:

$$[K]\{\partial U/\partial X_i\} = \{\partial P/\partial X_i\} \tag{5}$$

其右端项可由 (4) 式求出, 通过解上式的有限元方程, 可解出节点位移对各基本随机变量 $X_i$ 的导数 $\partial\{U\}/\partial X_i$ $(i=1, 2\cdots n)$。

应力公式可由 $\{\sigma\} = [D][B]\{U\}$ 求出, 由于弹性矩阵 $[D]$ 中的正交异性材料各物理量与几何矩阵 $[B]$ 中的几何形状参数与压力随机量无关, 所以应力对各压力随机量的导数可写为:

$$\partial\{\sigma\}/\partial X_i = [D][B](\partial\{U\}/\partial X_i) \quad (i=1,2\cdots n) \tag{6}$$

如已知位移导数, 则可求出应力导数。

3. 临界函数及其导数　对于定向结晶叶片材料, 按 Hill 准则写出临界函数

$$g = 1 - \frac{(\sigma_X - \sigma_Y)^2}{\sigma_T^2} - \frac{\sigma_X\sigma_Y + \sigma_Z^2 - \sigma_X\sigma_Z - \sigma_Y\sigma_Z}{\sigma_D^2} - \frac{\tau_{XY}^2}{\tau_{TT}^2} - \frac{\tau_{YZ}^2 + \tau_{XZ}^2}{\tau_{TD}^2} \tag{7}$$

$Z$ 为结晶方向, $X$、$Y$ 为非结晶方向; $\sigma_T$ 为非结晶方向材料的纯拉强度, $\sigma_D$ 为结晶方向的纯拉强度, $\tau_{TD}$ 为结晶—非结晶方向纯剪强度, $\tau_{TT}$ 为非结晶—非结晶方向的纯剪强度。

临界函数对各基本随机量的导数为:

$$\frac{\partial g}{\partial X_i} = \left(-2\frac{\sigma_X - \sigma_Y}{\sigma_T^2} - \frac{\sigma_Y - \sigma_Z}{\sigma_D^2}\right)\frac{\partial\sigma_X}{\partial X_i} + \left(2\frac{\sigma_X - \sigma_Y}{\sigma_T^2} - \frac{\sigma_X - \sigma_Z}{\sigma_D^2}\right)\frac{\partial\sigma_Y}{\partial X_i} + \left(\frac{\sigma_X + \sigma_Y - 2\sigma_Z}{\sigma_D^2}\right)\frac{\partial\sigma_Z}{\partial X_i}$$
$$- 2\frac{\tau_{XY}}{\tau_{TT}^2}\cdot\frac{\partial\tau_{XY}}{\partial X_i} - 2\frac{\tau_{Yz}}{\tau_{TD}^2}\cdot\frac{\partial\tau_{Yz}}{\partial X_i} - 2\frac{\tau_{ZX}}{\tau_{TD}^2}\cdot\frac{\partial\tau_{ZX}}{\partial X_i} \tag{8}$$

利用 (6) 式导出各应力分量对各随机变量的导数值代入 (8) 式即可求出 $\partial g/\partial X_i$。

4. 可靠度的计算　临界函数 $g(x)$ 在破坏点的展开式为:

$$g(x) = g(x)_* + \sum_{i=1}^{n}(X_i - X_i^*)\left(\frac{\partial g(x)}{\partial X_i}\right)_*$$

注标 "*" 表示为破坏点上的值

$$E[g(x)] = g(x)_* + \sum_{i=1}^{n}(E[X_i] - X_i^*)\left(\frac{\partial g}{\partial X_i}\right)_* \tag{9}$$

因各随机量已假定为独立无关, 其方差为:

$$V_{ar}[g(x)] = \sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i}\right)_* V_{ar}[X_i] \tag{10}$$

可靠性指标

$$\beta = E[g]/(V_{ar}[g])^{1/2} \tag{11}$$

当 $\beta$ 已知, 即可求其破坏概率 $P_f = \Phi(-\beta)$ 及可靠度 $Re = \Phi(\beta)$　　(12)

$\Phi(\ \ )$ 为标准正态分布的累积分布函数。

5. 计算步骤　(1) 利用 Hasofer-Lind 迭代计算, 由于破坏点值未知, 取各基本随机变量的均值为计算的初值。(2) 形成刚度矩阵。(3) 形成载荷阵 $\{P\}$ 及 $\partial\{P\}/\partial X_i(i=1,2\cdots n)$。(4) 置入位移边界条件, 计算 $\partial\{U\}/\partial X_i$ 时置入位移导数的边界条件。(5) 解出节点位移 $\{U\}$ 及 $\partial\{U\}/\partial X_i(i=1,2\cdots n)$。(6) 求出 $\{\sigma\}$ 及 $\partial\{\sigma\}/\partial X_i(i=1,2\cdots n)$。(7) 求出 $\partial g/\partial X_i(i=1,

$2\cdots n$)。(8) 导出 $E[g]$、$V_{ar}[g]$ 及 $\beta$ 值。(9) 选出 $\beta$ 值最小的单元和节点号，如与上次计算的节点相同且两次 $\beta$ 值之差小于预定精度要求，表明已逼近破坏点，否则利用

$$\alpha = \frac{\left(\frac{\partial g}{\partial X_i}\right)_* (V_{ar}[X_i])^{1/2}}{\left[\sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i}\right)_*^2 \cdot V_{ar}[X_i]\right]^{1/2}} \quad 及 \quad X_i = E[X_i] - \alpha\beta\sqrt{V_{ar}[X_i]}$$

求得新破坏点，返回 (3) 重新计算。

6. 随机有限元程序的验证　本方法及程序，计算可靠度是否合理，应予验证，但用实验的方法是很困难的，故选用几个与 $[k]$ 阵中的物理量无关的随机变量采用模拟计算方法校核。如果用 Monte-Carlo 模拟去解有限元方程，则耗机时巨大，在一般计算机上不太可能实现，本文选用可写出解析表达式的 $\rho$、$\omega$ 为随机变量进行 Monte-Carlo 模拟计算，同时再用随机有限元程序去计算其可靠性指标，二者进行对比，以证实本方法及程序的正确性。

应用 Monte-Carlo 模拟计算时，考虑到本例是等温状态所以其应力 $\{\sigma\} \propto \rho\omega^2$

利用随机系数 $\rho_o$、$\omega_o$ 写出临界函数的解析表达式:

$$g = 1 - (\rho_o\omega_o^2)^2\left\{\frac{(\sigma_X - \sigma_Y)^2}{\sigma_T^2} + \frac{\sigma_X\sigma_Y + \sigma_z^2 - \sigma_X\sigma_z - \sigma_Y\sigma_z}{\sigma_D^2} + \frac{\tau_{XY}^2}{\tau_{TT}^2} + \frac{\tau_{Yz}^2 - \tau_{zX}^2}{\tau_{TD}^2}\right\} \tag{13}$$

$\rho_o$、$\omega_o$ 均值为 1，离差系数分别等于 $\rho$、$\omega$ 二个随机变量的离差系数。

应用随机有限元计算，其载荷列阵可写成下式:

$$\{P_i\}^e_{\rho\omega} = \iiint N_i\{r\}\rho\omega^2 dXdYdz \tag{14}$$

$\{r\}$ 为旋转中心到单元内微元体的矢量在三个座标轴上的投影，其载荷对随机量的导数为:

$$\frac{\partial\{P_i\}^e_{\rho\omega}}{\partial\rho} = \iiint N_i\{r\}\omega^2 dXdYdz = \frac{1}{\rho}\{P_i\}^e_{\rho\omega} \tag{15}$$

$$\frac{\partial\{P_i\}_{\rho\omega}}{\partial\omega} = \iiint N_i\{r\}2\rho\omega dXdYdz = \frac{2}{\omega}\{P_i\}^e_{\rho\omega} \tag{16}$$

有了上式后，代入有限元程序，按上述方法及步骤，即可求出可靠性指标。

| 迭代次数 | $\beta$值 | 注 |
|---|---|---|
| 1 | 33.368335 | 均值点结果 |
| 2 | 27.807146 | |
| 3 | 27.661944 | |
| 4 | 27.658582 | 破坏点结果 |

计算结果表明，两种方法计算之可靠度相差 0.112%，这说明本文的计算方法及编制的随机有限元程序是可信的。

7. 算例　本例计算为某发动机涡轮叶片，材料数据选用 DZ-22 材料说明，压力数据由本校 704 教研室提供。在压力随机量暂选定离差系数为 0.1 时，经四次迭代 $\beta$ 值已经稳定，计算结果如下:

当 $\beta = 3.1$ 时 $Re = 0.999032$；$\beta = 3.99$ 时 $Re = 0.999967$，上述 $\beta$ 值相当大，已查不出相对应的可靠度，与其它随机量相比，压力场的随机变化，对叶片的可靠度影响很小。

## 参　考　文　献

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# A RESEARCH ON CRACKED FAILURES OF FIR-TREE SERRATION IN AEROENGINE TURBINE DISC

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**ABSTRACT** A lot of cracked failures of fir-tree serration accurred in the second stage turbine disc of a large numbers of certain types aeroengines in service. The characte of these cracks belongs to mechanical fatigue. In order to find out the cause for the cracked failure, the vibrating stresses of the second stage turbine blades on test bench and their vibrating strains in flight tests have been measured in the conditions of high temperature and high rotational speed. The measured data indicate that several resonance zones really exist in the second stage turbine blades of type A and B engines. Some methods for elimination of the cracked failures of fir-tree serration in the turbine disc are provided. According to the experiece in service, a modification of type B engine hase eliminated the cracked failures of fir-tree serration of turbine disc in its service life.

# RELIABILITY ANALYSIS OF DIRECTIONAL SOLIDIFICATION BLADE

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**ABSTRACT** The stochastic Finite Element Method and Hasofer-Lind Method are adopted to analyse the reliablity influence of stochastic pressure field on the directional solidification blade. This method and program are verified by calculation of the Monte-Carlo Method.

# AN EFFICIENT 2D-GRID GENERATION APPROACH USING ARC-LENGTH AS A PARAMETER

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**ABSTRACT** A fast and efficient grid generation program has been developed for any two-dimensional configurations. In this scheme, the separate grids are generated in each zone with a high speed algorithm which makes the ratio of the coordinate increments satisfy a linear distribution along arc-length of a grid curve, and then they smoothly joined along the common boundaries to establish the grid for the complex configuration. The advantages of the scheme are as follows: First, the mesh lines have continuous slopes from one zone to another; Second, the degree of the mesh lines clustering can be well controlled by the distribution of the boundary points; And the third, the computational speed is ten and even several hundred times faster than the elliptic grid generation scheme which has been widely used nowadays.