

裂纹转子通过参数共振区的瞬态特性

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【摘要】 采用主谐波变换将通常的裂纹转子的运动微分方程变为了一种即可求定常运动,又便于研究非定常运动的方程。分别针对理想和非理想能源研究了裂纹转子加速通过参数共振区的瞬态振动特性。据此提出,转子在 $2\omega/3$ 和 2ω 附近的瞬态振动可能成为诊断裂纹的一种重要依据。

关键词: 裂纹转子 参数共振 瞬态振动 监测

1 前 言

有关裂纹转子动力特性的研究越来越受到人们的重视^[1]。目前,一些学者研究了裂纹转子过临界转速区的瞬态特性^[2],为象航空发动机这样的经常起动、停机的旋转机械的裂纹诊断提供了一定的理论依据。由于其它因素如支承非线性等对转子在临界转速区的运动也可能有影响,因此,仅仅采用过临界转速区的瞬态特性,就难以区分出这是否是由于裂纹造成的。最近,我们已经发现:裂纹转子在 $2\omega/3$ 、 2ω (ω 为无裂纹时的临界转速)附近存在着参数共振现象^[3],而且在参数共振区转子的运动是不稳定的,那么当转子加速或减速通过这些区域时,又会怎样运动?这些运动特性能否被用来诊断裂纹?在国内外文献中尚未发现有关这方面的研究,而这正是本文要研究的问题。

2 理论推导

对于固支 Jeffcott 转子,设刚性转盘位于两支承中央,转盘附近的轴上有一横向弓形疲劳裂纹,不考虑转盘的回转效应及轴的分布质量。当能源是非理想时,根据文献[2、4]裂纹转子系统在重力及不平衡激励下无量纲形式的运动微分方程是:

$$\begin{aligned} X'' + \zeta \dot{X} + X - 0.5\epsilon(1 - \cos\varphi)(\delta - \cos 2\varphi X - \sin 2\varphi) &= -E_1\varphi^2 \sin(\varphi - \beta) \\ Y'' + \zeta \dot{Y} + Y - 0.5\epsilon(1 - \cos\varphi)(-\sin 2\varphi X + (\delta + \cos 2\varphi)Y) &= -G + E_1\varphi^2 \cos(\varphi - \beta) \end{aligned} \quad (1)$$

$$\begin{aligned} \varphi'' &= (1/2)\rho^2(R_T - S\varphi - Q\dot{\varphi}) + 0.5\rho^2 \dot{X} \sin(\varphi - \beta) - Y'' \cos(\varphi - \beta) \\ &\quad + 0.5\rho^2 \dot{\varphi} \dot{X} \cos(\varphi - \beta) - Y' \sin(\varphi - \beta) \end{aligned}$$

其中: $X = x/l$, $Y = y/l$, $E_1 = e/l$, $\Omega = \omega/\omega$, $\tau = \omega t$, $\varphi = c/l(m\omega)$, $\rho = e - ml/J_d$, $G = 9.8/(l\omega^2)$; x, y 为盘心横向位移坐标, e 为质量偏心距, ω 为转子的转速, ω 为无裂纹时临界转速, t 为时间, c 为横向阻尼系数, m 为转盘质量, J_d 为转盘赤道转动惯量, S, Q 为能源内部阻力系数, R_T 为能源驱动力矩, β 为裂纹方向与偏心方向的夹角, l 为长度量纲的量, “ $'$ ”表示对 τ 的导数, ϵ, δ 由裂纹深度来决定(关系见文献[4])。对于理想能源

$$\dot{\varphi} = a(\text{常数}) \quad (2)$$

许多文献的稳态计算表明盘心的轨迹中主谐波多占主要成份,且其轨迹多可近似为椭圆,因而我们假设方程(1)的解具有下列形式:

$$\begin{aligned} X &= x_c(\tau) \cos(\mathcal{Q}\tau - \beta) - x_s(\tau) \sin(\mathcal{Q}\tau - \beta) \\ Y &= y_c(\tau) \cos(\mathcal{Q}\tau - \beta) - y_s(\tau) \sin(\mathcal{Q}\tau - \beta) - G \end{aligned} \quad (3)$$

这样,当轨迹为主谐波下的椭圆时,(3)式系数为常数,而当轨迹为其它形状时,(3)式仍将适用,只是系数为 τ 的函数而已。将(3)式代入(1),并考虑补充条件 $x'_c \cos(\varphi - \beta) - x'_s \sin(\varphi - \beta) = 0$, $y'_c \cos(\varphi - \beta) - y'_s \sin(\varphi - \beta) = 0$,求得解:

$$\begin{aligned} x'_c &= A \sin(\varphi - \beta), \quad x'_s = A \cos(\varphi - \beta), \quad y'_c = B \sin(\varphi - \beta), \quad y'_s = B \cos(\varphi - \beta) \\ \dot{\varphi} &= 0.5\rho^2(R\tau - S\dot{\varphi} - Q\dot{\varphi}) + 0.5\rho^2\mathcal{Q}\alpha_1 [\zeta \sin(\varphi - \beta) - \mathcal{Q} \cos(\varphi - \beta)] \\ &\quad - 0.5\rho^2\mathcal{Q}\alpha_2 [\zeta \cos(\varphi - \beta) - \mathcal{Q} \sin(\varphi - \beta)] - 0.5\rho^2\alpha_2 \zeta \sin(\varphi - \beta) \\ &\quad - 0.5\epsilon \sin(\varphi - \beta) (1 - \cos\mathcal{Q}(\delta - \cos 2\mathcal{Q}) - 0.5\epsilon \cos(\varphi - \beta) (1 - \cos\mathcal{Q} \sin 2\mathcal{Q}) \\ &\quad + 0.25\rho^2 \epsilon G (1 - \cos\mathcal{Q}) \zeta \sin(\varphi - \beta) \sin 2\mathcal{Q} + \cos(\varphi - \beta) (\delta + \cos 2\mathcal{Q})] \\ &\quad + 0.5\rho^2\alpha_2 \zeta \cos(\varphi - \beta) - 0.5\epsilon \sin(\varphi - \beta) (1 - \cos\mathcal{Q} \sin 2\mathcal{Q}) \\ &\quad - 0.5\epsilon \cos(\varphi - \beta) (1 - \cos\mathcal{Q}(\delta + \cos 2\mathcal{Q})) - 0.5\rho^2 E_1 \dot{\varphi} \end{aligned} \quad (4)$$

式中

$$\begin{aligned} A &= \{-\mathcal{Q}\alpha_1 - \mathcal{Q}\alpha_2 - \zeta\dot{\alpha}_1 + \alpha_2 - 0.5\epsilon(1 - \cos\mathcal{Q})(\delta - \cos 2\mathcal{Q})\alpha_2 \\ &\quad - \sin 2\mathcal{Q}\alpha_2 - G\} + E_1 \dot{\varphi} \sin(\varphi - \beta) / \dot{\varphi} \\ B &= \{-\mathcal{Q}\alpha_2 - \mathcal{Q}\alpha_1 - \zeta\dot{\alpha}_2 + \alpha_1 - 0.5\epsilon(1 - \cos\mathcal{Q})(\delta + \sin 2\mathcal{Q})\alpha_1 \\ &\quad + (\delta + \cos 2\mathcal{Q})(\alpha_1 - G)\} - E_1 \dot{\varphi} \cos(\varphi - \beta) / \dot{\varphi} \\ \alpha &= x_c \sin(\varphi - \beta) + x_s \cos(\varphi - \beta), \quad \alpha_2 = x_c \cos(\varphi - \beta) - x_s \sin(\varphi - \beta) \\ \alpha_1 &= y_c \sin(\varphi - \beta) + y_s \cos(\varphi - \beta), \quad \alpha_2 = y_c \cos(\varphi - \beta) - y_s \sin(\varphi - \beta) \end{aligned}$$

这样裂纹转子系统的运动微分方程(1)由于采用主谐波变换而变为在主谐系数表示形式下的微分方程组(4)。通过该方程组可以直接得到激励频率的包络线。方程(4)与方程(1)是完全等价的,但方程(4)不仅可求定常运动,而且便于研究非定常运动。另外,在对(4)式进行平均之后,可仅通过代数方程组来求转子近似稳态轨迹,这种方法的优点是方便、省时。式(4)对应于非理想能源。而对于理想能源情况,则(2)式与(4)式前4个方程联立。

3 数值分析

我们的研究结果已表明裂纹转子运动存在着参数共振现象,且在参数共振的转速附近,存在不稳定区域(即参数共振区),也就是说转子的定常运动在此区域趋于无穷。这里所要分析的则是当裂纹转子加速通过参数共振区的非定常运动。通过对所推方程求数值解,可得裂纹转子主谐激励振动包络线($x_c^2 + x_s^2$ 或 $y_c^2 + y_s^2$)随转速的变化。

图1是驱动能源为理想的情况下,裂纹对转子在 $\Omega \approx 2$ 附近瞬态响应的影响。由图可知,转子的瞬态运动对裂纹相当敏感,无裂纹时,曲线形状变化平缓。而当相对裂纹深度 $\mu = 0.05$ 时,曲线变化则明显紊乱,而且在图示转速区域中,曲线的最大值随着裂纹的出现而非常明显的增大;图2为驱动能源非理想的情况下,裂纹深度对转子在 $\Omega \approx 2/3$ 附近瞬态响应的影响。不同的裂纹深度相比,虽然曲线的形状非常相似,但是其最大幅值却不相同,当 $\mu = 0.05$ 时,

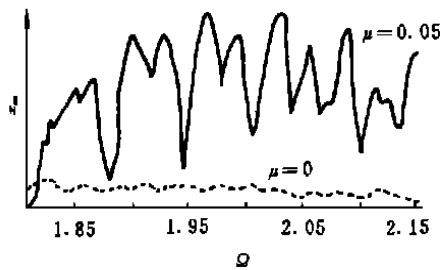


图 1 理想能源下转子瞬态运动幅值—转速图

(a = 0.01)

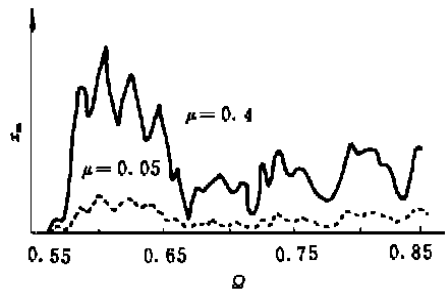


图 2 非理想能源下转子瞬态运动幅值—转速图

(R_T = 10000, S = 3000, Q = 0)

最大幅值为 7.211×10^{-3} , 当 $\mu = 0.4$ 时, 最大幅值为 3.064×10^{-2} , 相差达 4 倍多。为明确起见, 我们在图 1、2 中仅分别给出了两条曲线。实际上, 图 1 对应情况所具有的特性对图 2 的情况也适用, 反之亦然。

图 3 是加动能源为理想的情况下, 加速度对裂纹转子过 $\Omega \approx 2$ 附近时瞬态响应的影响, 由图可知: 在同样裂纹深度下裂纹转子的瞬态幅值将随加速度的增而加减小。

4 结 论

(1) 不论能源是否理想, 有裂纹时, 转子在 $2\omega/3$ 和 2ω 附近的瞬态振动的最大幅值一般要比无裂纹时的大; 当相对裂纹深度 $\mu < 0.4$ 时, 这样的最大幅值将随裂纹深度的增大而增大。

(2) 对于理想能源, 当加速度增大时, 同样裂纹深度的转子在参数共振区的最大瞬态振动幅值将随之减小。

由于参数共振是裂纹转子所特有的性质, 综合以上特性, 本文提出转子在 $2\omega/3$ 和 2ω 附近的瞬态振动可能成为诊断裂纹的一种重要依据。

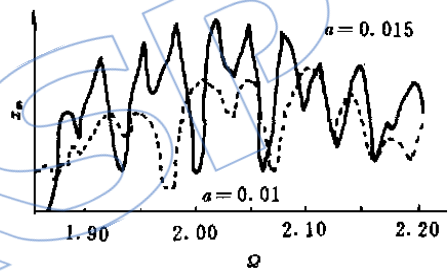


图 3 理想能源下转子瞬态运动幅值—转速图

(μ = 0.4)

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FINITE ELEMENT ANALYSIS AND EXPERIMENTS ON CREEP OF MULTIHOLED PLATE DZ-22

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ABSTRACT

Experiments on tension creep behavior of the ortho-anisotropic material DZ-22 have been performed at 750°C, 850°C and 900°C. This material is applied to directionally-solidified air-cooled turbine blade. A constitutive equation of the creep is formulated by fitting with the experimental data. The equation shows three stages of the creep behavior. Effectiveness of the FEM analysis for the creep behavior of the ortho-anisotropic material is proved by the experimental results. Then, in consideration of the features of the air-cooled blade, a simplified FEM is presented for analysis of the creep of a multiholed component and is proved by the creep experiments with densely multiholed plate specimens.

TRANSIENT BEHAVIOUR OF CRACKED ROTOR THROUGH PARAMETRIC RESONANCE RANGE

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ABSTRACT

In order to diagnose the existence of a crack in rotating machinery, the transient vibration of a cracked rotor through parametric resonance range is investigated. By means of primary harmonic transformation, the normal motion equation of the cracked rotor is transformed into a new differential one. From direct solution of this equation, the amplitude of primary harmonic vibration can be obtained under nonstationary or stationary excitation. In respective consideration of ideal and nonideal sources of energy, it is found that: (1) Whether the source of energy is ideal or not, the largest amplitude of transient vibration near $2\omega/3$ and 2ω (Parametric resonance range) of the rotor with crack is bigger than that without crack (where ω is the critical speed of the rotor without crack). And the deeper the crack, the larger the amplitude in the case of the ratio of crack depth to shaft diameter less than 0.4. (2) When the source of energy is ideal, the larger the acceleration, the less the largest amplitude of transient vibration through parametric resonance ranges of the cracked rotor. Because parametric resonance is the exclusive characteristic of cracked rotor according to the authors' previous work, the authors believe that the transient vibration behaviors near $2\omega/3$ and 2ω may be a basis of diagnosing cracks on a shaft of rotating machinery.

A NEW METHOD FOR EXTENDING VECTOR EQUATIONS OF VISCOUS FLOW IN TURBINE IN CURVILINEAR COORDINATES

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ABSTRACT

A new idea is suggested to extend the viscous vector equations in non-orthogonal curvilinear coordinates. In the expansion equation obtained with the help of this new method, the number of terms is greatly decreased, so that it is easy to apply to engineering calculation. Meanwhile the expanded forms of compressible and incompressible equations are also presented and a generalized expression is given. Finally, by the aid of this new method, the numerical calculations have been carried out for 2-dimensional compressible viscous cascade flow and 3-dimensional compressible viscous cascade flow. It is shown that the new expansion equation is of great value to engineering practice.