

叶轮机机械气动声学频域分析*

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【摘要】 得到中弧线小弯度薄叶片叶轮机机械声场的详细结果。讨论适用于近声场, 叶片的弯、掠等几何特征及运行条件也都在频域解中显式地表示出来。

主题词: 气动声学 叶轮机机械 频域分析

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1 旋转源声场频域解

在叶轮机机械气动声学研究方面较早的工作是讨论紧凑源远场问题, 这对深入了解此类物理现象的机理及其声学设计是远远不够的。文献[1]得到旋转源近声场的频域解, 本文将之应用于叶轮机机械近声场研究(以下各符号除非特殊说明均见文献[1]), 所得结果显式地表示了叶片径向、轴向位置非紧凑性对声场结构的影响。

强度分布为 $a(\bar{Y}, \tau)$ 的源其声压时域积分解为:

$$p(\bar{X}, t) = \int_{-\infty}^{\infty} \iiint a(\bar{Y}, \tau) G d\bar{Y} d\tau, \text{ Green 函数: } G = \delta(t - \tau - r_s / C_0) / 4\pi r_s \quad (1)$$

设源绕 y_1 轴旋转(见文献[1]之图1), 那么其声场频域解为^[1]:

$$p(\bar{X}, \omega) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} p_n(\bar{X}, \omega') \delta(\omega - \omega' - n\Omega) d\omega' \quad (2)$$

其中 n 阶声压复系数由 4 部分组成: $p_n = p_{n0} + p_{n1} + p_{n2} + p_{n3}$ 。为书写方便略写自变量, 并令 $Q = e^{ik_0 r} / 4\pi r$, $\lambda = k_0 R \sin \theta$, 那么远场解 p_{n0} 、轴向位置 y_1 引起的近场修正 p_{n1} 应分别为:

$$p_{n0} = Q \iiint_{(y_0)} J_n(\lambda) e^{-i[n(\theta_b + \pi/2) + k_0 y_1 \cos \theta]} A(\bar{\zeta}, \omega) d\bar{\zeta}$$

$$p_{n1} = \frac{Q}{r} \iiint_{(y_0)} y_1 \cos \theta J_n(\lambda) e^{-i[n(\theta_b + \pi/2) + k_0 y_1 \cos \theta]} A(\bar{\zeta}, \omega) d\bar{\zeta}$$

径向位置 R 、周向位置 θ 引起的近场修正为:

$$p_{n2} = \frac{Q}{r} \iiint_{(y_0)} \frac{-2}{2} R \sin \theta \cos(\theta - \pi/4) J_{n+1}(\lambda) e^{-i[(n+1)(\theta_b + \pi/2) + k_0 y_1 \cos \theta]} A(\bar{\zeta}, \omega) d\bar{\zeta}$$

$$p_{n3} = \frac{Q}{r} \iiint_{(y_0)} \frac{-2}{2} R \sin \theta \cos(\theta + \pi/4) J_{n-1}(\lambda) e^{-i[(n-1)(\theta_b + \pi/2) + k_0 y_1 \cos \theta]} A(\bar{\zeta}, \omega) d\bar{\zeta}$$

2 叶轮机机械气动噪声源

只考虑线性源(厚度源与载荷源),作薄叶片假设,叶片法向速度 V_n 及作用力 f_i 分别为叶片两侧相应点处的合法向速度和合力。中弧线中点柱坐标为 (R, RM, FA) , σ 为倾角。叶片中弧线坐标 ϵ 其原点在弧线中点。径向力未做讨论。在叶面上积分式(1)有:

$$p(\bar{X}, t) = - \int_{-\infty}^{\infty} \int \int \left[\rho V_n \frac{\partial G}{\partial \tau} + f_i \frac{\partial G}{\partial y_i} \right] \frac{1}{|n_i|} R dR d\theta d\tau \quad (3)$$

n_i : 叶片面法向矢量的轴向投影。设有厚度分布函数 $h(\epsilon)$, 其规范化函数 $H(Z) \in [0, 1]$, $Z \in [-1/2, 1/2]$, $h(\epsilon) = b t_b H(Z)$, $Z = \epsilon / b$ (b : 弦长, t_b : 最大厚度与弦长之比)。用上加“ $\sim$ ”表示随体坐标中的量值, 那么厚度体分布函数为: $H(R, \theta - \Omega\tau, y_1) = h(\epsilon) \delta(y_1 - y_1^c)$, 由此得到:

$$V_n(\bar{Y}, \tau) = R \Omega \sin \sigma \int_{-\infty}^{\infty} H(R, \theta - \Omega\tau, y_1) dy_1, \quad H(R, \theta - \Omega\tau, y_1) = \partial H(R, \theta - \Omega\tau, y_1) / \partial \epsilon$$

非定常问题力分布函数为 $f_i(\epsilon, \tau)$, 其体分布函数为 $F_i(\bar{Y}, \tau) = f_i(\epsilon, \tau) \delta(y_1 - y_1^c)$ 。规范化函数:

$$E_1(Z, \tau) = f_1(\epsilon, \tau) \left| \frac{1}{2} \rho R^2 \Omega^2 C_1 \right|, \quad C_1 = \int_{-\infty}^{\infty} \int_{b/2}^{\pi/2} f_1 d\epsilon d\tau \left| \frac{1}{2} \rho R^2 \Omega^2 \right|$$

$$E_\theta(Z, \tau) = f_\theta(\epsilon, \tau) \left| \frac{1}{2} \rho R^2 \Omega^2 C_\theta \right|, \quad C_\theta = \int_{-\infty}^{\infty} \int_{b/2}^{\pi/2} f_\theta d\epsilon d\tau \left| \frac{1}{2} \rho R^2 \Omega^2 \right|$$

将上述厚度、力分布函数代入式(3)并与(1)比较可得到叶轮机机械厚度源和力源分别为

$$a_T = \rho R \Omega \sin \sigma \bar{H}(R, \theta - \Omega\tau, y_1) \partial \epsilon / \partial \tau, \quad a_F = \partial f_i(\bar{Y}, \tau) / \partial y_i$$

3 叶轮机机械声辐射频域解

叶片中弧线弯度较小时近似有 $\epsilon = R(\theta_b - RM) / \sin \sigma$, $\theta_b = RM + \epsilon \sin \sigma / R$, $y_1^c = FA + \epsilon \cos \sigma$ 。

将前述噪声源表达式代入(2)可以得到下述分别对应于 $P_{n0}, P_{n1}, P_{n2}, P_{n3}$ 的解。

$$\begin{bmatrix} p_{n0T} \\ p_{n0\theta} \\ p_{n01} \end{bmatrix} = \rho \Omega^2 Q \int R^2 J_n(\lambda) e^{-i(\mathcal{Q}_0 + n\mathcal{Q}_s)} \begin{bmatrix} k_{x0}^2 t_b \sin \sigma \Psi_{0T}(k_{x0}) \delta(\omega) \\ i k_{x0} (C_\theta / 2) \Psi_{0\theta}(k_{x0}, \omega) \\ i k_{y0} \sin \sigma (C_1 / 2) \Psi_{01}(k_{x0}, \omega) \end{bmatrix} dR \quad (4)$$

下标 $T, \theta, 1$ 分别表示厚度源、周向和轴向力源。 $\mathcal{Q}, \mathcal{Q}_s$ 分别表达了叶片轴向和周向位置变化产生的相移: $\mathcal{Q} = k_0 \cdot FA \cdot \cos \sigma$, $\mathcal{Q}_s = RM + \pi / 2$ 。 k_{x0}, k_{y0} 为波数, $\Psi_{0T}, \Psi_{0\theta}, \Psi_{01}$ 则为波形状函数:

$$k_{x0} = b(n \sin \sigma / R + k_0 \cos \theta \cos \sigma), \quad k_{y0} = b k_0 \cos \theta, \quad k_{x1} = b[(n+1) \sin \sigma / R + k_0 \cos \theta \cos \sigma]$$

$$k_{x2} = b[(n-1) \sin \sigma / R + k_0 \cos \theta \cos \sigma], \quad \Psi_{0T}(k_{x0}) = \int_{-1/2}^{1/2} H(Z) e^{-ik_{x0}Z} dZ,$$

$$\Psi_{0\theta}(k_{x0}, \omega) = \int_{-1/2}^{1/2} E_\theta(Z, \omega) e^{-ik_{x0}Z} dZ, \quad \Psi_{01}(k_{x0}, \omega) = \int_{-1/2}^{1/2} E_1(Z, \omega) e^{-ik_{x0}Z} dZ$$

$$\begin{bmatrix} p_{n1T} \\ p_{n1\theta} \\ p_{n11} \end{bmatrix} = - \frac{\rho \Omega^2 Q}{r} \int R^2 J_n(\lambda) e^{-i(\mathcal{Q}_0 + n\mathcal{Q}_s)} \begin{bmatrix} b t_b \sin \sigma \cos \theta (-ik_{x0}) \Psi_{1T}(k_{x0}) \delta(\omega) \\ b (C_\theta / 2) \Psi_{1\theta}(k_{x0}, \omega) \\ b \sin \sigma \cos \theta (C_1 / 2) \Psi_{11}(k_{x0}, \omega) \end{bmatrix} dR \quad (5)$$

$$\Psi_{1T}(k_{x0}) = \int_{-1/2}^{1/2} [2 \cos \sigma - ik_{x0}(FA/b + Z \cos \sigma)] H(Z) e^{-ik_{x0}Z} dZ$$

$$\Psi_{1\theta}(k_{x0}, \omega) = \int_{-1/2}^{1/2} [\cos \sigma - ik_{x0}(FA/b + Z \cos \sigma)] E_\theta(Z, \omega) e^{-ik_{x0}Z} dZ$$

$$\Psi_{11}(k_{x0}, \omega) = \int_{-1/2}^{1/2} [1 - ik_{x0} \cos \theta (FA/b + Z \cos \sigma)] E_1(Z, \omega) e^{-ik_{x0}Z} dZ$$

$$\begin{bmatrix} p_{n2T} \\ p_{n2\theta} \\ p_{n21} \end{bmatrix} = \frac{\rho \Omega^2 Q}{r} \int \frac{\overline{2}}{2} R^3 J_{n+1}(\lambda) e^{-i[\varphi_0 + (n+1)\varphi_s]} \begin{bmatrix} t b \sin \sigma \sin \theta \Psi_{2T}(k_{x1}) \delta(\omega) \\ (C\theta/2) \Psi_{2\theta}(k_{x1}, \omega) \\ i k_{y0} \sin \sigma \sin \theta (C1/2) \Psi_{21}(k_{x1}, \omega) \end{bmatrix} dR \quad (6)$$

令 $\eta = b \sin \sigma Z / R + RM$, 则

$$\Psi_{2T}(k_{x1}) = \int_{-1/2}^{1/2} \cos \left[\eta - \frac{\pi}{4} \right] \left[\frac{b^2 \sin^2 \sigma}{R^2} - 2 i k_{x1} \frac{b \sin \sigma}{R} \operatorname{tg} \left[\eta - \frac{\pi}{4} \right] + k_{x1}^2 \right] \cdot H(Z) e^{-i k_{x1} Z} dZ$$

$$\Psi_{2\theta}(k_{x1}, \omega) = \int_{-1/2}^{1/2} \left[\frac{b \sin \sigma}{R} \sin \left[\eta - \frac{\pi}{4} \right] + i k_{x1} \cos \left[\eta - \frac{\pi}{4} \right] \right] \cdot E_{\theta}(Z, \omega) e^{-i k_{x1} Z} dZ$$

$$\Psi_{21}(k_{x1}, \omega) = \int_{-1/2}^{1/2} \left[\cos \left[\eta - \frac{\pi}{4} \right] \right] E_1(Z, \omega) e^{-i k_{x1} Z} dZ$$

$$\begin{bmatrix} p_{n3T} \\ p_{n3\theta} \\ p_{n31} \end{bmatrix} = \frac{\rho \Omega^2 Q}{r} \int \frac{\overline{2}}{2} R^3 J_{n-1}(\lambda) e^{-i[\varphi_0 + (n-1)\varphi_s]} \begin{bmatrix} t b \sin \sigma \sin \theta \Psi_{3T}(k_{x2}) \delta(\omega) \\ (C\theta/2) \Psi_{3\theta}(k_{x2}, \omega) \\ i k_{y0} \sin \sigma \cos \theta (C1/2) \Psi_{31}(k_{x2}, \omega) \end{bmatrix} dR \quad (7)$$

$$\Psi_{3T}(k_{x2}) = \int_{-1/2}^{1/2} \cos \left[\eta + \frac{\pi}{4} \right] \left[\frac{b^2 \sin^2 \sigma}{R^2} - 2 i k_{x2} \frac{b \sin \sigma}{R} \operatorname{tg} \left[\eta - \frac{\pi}{4} \right] + k_{x2}^2 \right] \cdot H(Z) e^{-i k_{x2} Z} dZ$$

$$\Psi_{3\theta}(k_{x2}, \omega) = \int_{-1/2}^{1/2} \left[\frac{b \sin \sigma}{R} \sin \left[\eta + \frac{\pi}{4} \right] + i k_{x2} \cos \left[\eta - \frac{\pi}{4} \right] \right] \cdot E_{\theta}(Z, \omega) e^{-i k_{x2} Z} dZ$$

$$\Psi_{31}(k_{x2}, \omega) = \int_{-1/2}^{1/2} \left[\cos \left[\eta + \frac{\pi}{4} \right] \right] E_1(Z, \omega) e^{-i k_{x2} Z} dZ$$

4 几点讨论

(1) 解不但可以处理远场(p_{n0}), 亦可用于分析近场(p_{n1}, p_{n2}, p_{n3}); (2) 解清楚地表达了厚度、周向力和轴向力各自引起的声场, 这为分析提供了方便; (3) 作薄叶片、中弧线小弯度假设, 但整个解中考虑了叶片的弯、掠, 因而适用性较广; (4) 由(4)式可知, 弯、掠在远场只以 φ_0 、 φ_s 形式影响声场相位, 但在近场(式 5, 6, 7) 不但影响相位还影响到幅值, FA 影响 p_{n1} , RM 影响 p_{n2} 、 p_{n3} ; (5) 除远场外, 所有波形状函数均为加权函数, 这些“权”体现了弯、掠的影响。

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SURVEY PAPER

PRELIMINARY SEARCH FOR NEW FLOW- TYPES IN TURBOMACHINERY

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ABSTRACT

As viewed from flow- types, the developing process of aircraft aerodynamics is compared with that of turbomachine gasdynamics. For the branch of turbomachine's gasdynamics, it is found that the steady body- fitted flow- type is still the searching object. However, the entire body- fitted flow- type is impossible. It means that the unexpected separations in turbomachinery can not be eliminated. And our comparison has shown that, as for the development of flow- types, the branch of turbomachine's gasdynamics lags considerably behind the branch of aircraft aerodynamics! Some of the suggestions, with regard to the possible ways of introducing new ideas of flow- types from aircraft aerodynamics into turbomachine gasdynamics, are presented in this paper.

Key words Turbomachines Vortex flow Non- steady flow Separated flow

FULL- LENGTH PAPERS

AEROACOUSTIC SOLUTION OF TURBOMACHINERY IN FREQUENCY DOMAIN

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ABSTRACT

According to the sound radiation solution of rotating source in frequency domain, formulae of noise sources in turbomachine have been derived. Then the detailed aeroacoustic solutions of the sound field in the turbomachine, the blades of which are thin and bent slight in the mid- chord alignment were obtained. The result is suitable for near sound field analysis. Moreover the influence of geometric parameters and operation conditions are explicitly expressed in the frequency domain solutions.

Key words Turbomachines Frequency domain analysis Aerodynamic noise

EFFECT OF SMALL RADIAL DISTORTION OF INLET FLOW ON STALL FLUTTER IN AXIAL FLOW COMPRESSOR

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ABSTRACT

The effect of small radial stagnation pressure distortion of inlet