

计算滑动轴承动特性系数的有限元法

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【摘要】 提出了一种快速求解 8 个轴承动特性的系数有限元法。该方法不仅比以往的有限元法计算速度快, 而且与其他方法具有同样的计算精度。

主题词: 有限元法 矩阵 滑动轴承
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求解轴承动特性系数数占全部轴承参数计算的大部分时间^[1]。本文根据 Reynolds 方程的有限元方程, 只需求解一次 Reynolds 方程便可得到 8 个动特性系数。采用四边形单元不仅较好地反映了该方法的一般性和通用性, 而且在其他条件相同的情况下, 可以用较少的网格获得用三角形单元需较多网格才能达到的精度, 这对节省计算时间是大有好处的^[2]。

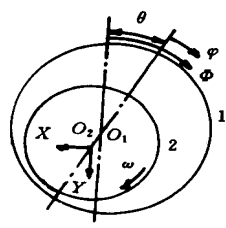
1 油膜压力的求解

静态 Reynolds 方程无量纲形式为

$$\frac{\partial}{\partial \varphi} \left(h^3 \frac{\partial p}{\partial \varphi} \right) + \alpha^2 \frac{\partial}{\partial \lambda} \left(h^3 \frac{\partial p}{\partial \lambda} \right) = 3 \frac{\partial h}{\partial \varphi} \quad (1)$$

$$p|_{\varphi=\varphi_a, \varphi=\varphi_b, \lambda=1} = 0, \quad \left(\frac{\partial p}{\partial \varphi} \right)|_{\varphi=\varphi_a, \varphi=\varphi_b} = 0$$
$$\left(\frac{\partial p}{\partial \lambda} \right)|_{\lambda=1} = 0 \quad (2)$$

式中, p 为油膜压力; 油膜厚度 $h = 1 + \epsilon \cos \varphi$, ϵ 为偏心率, α 为轴承宽径比的倒数; φ_a, φ_b 为油膜完整区上、下游边界; φ, λ 为油膜周向和横向坐标。坐标系及边界条件如图 1、2 所示(图 1 中: 1 为轴承, 2 为轴颈)。



O₁ 轴承中心 O₂ 轴颈中心

图 1 坐标系分析

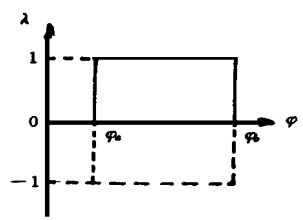


图 2 边界条件

Reynolds 方程的等价泛函, $L(p)$ 为

$$L(p) = - \iint h^3 \left[\left(\frac{\partial p}{\partial \varphi} \right)^2 + \alpha^2 \left(\frac{\partial p}{\partial \lambda} \right)^2 \right] d\varphi d\lambda - 6 \iint \frac{\partial h}{\partial \varphi} p d\varphi d\lambda \quad (3)$$

运用有限元法, 采用八节点四边形单元, 由变分原理知, Reynolds 方程的解 p 应满足

$$\frac{\partial L(p)}{\partial p_i} = \sum_{i=1}^E \frac{\partial L_e(p)}{\partial p_i} = 0$$

即

$$\frac{\partial L_e(p)}{\partial p_i} = - 2 \iint h^3 \left[\frac{\partial p}{\partial \varphi} \frac{\partial}{\partial p_i} \left(\frac{\partial p}{\partial \varphi} \right) + \alpha^2 \frac{\partial p}{\partial \lambda} \frac{\partial}{\partial p_i} \left(\frac{\partial p}{\partial \lambda} \right) \right] d\varphi d\lambda - 6 \iint \frac{\partial h}{\partial \varphi} \frac{\partial p}{\partial p_i} d\varphi d\lambda \quad (4)$$

式中, p_i 为第 i 个节点压力, e 为任意四边形单元, E 为单元总数。

将所有单元都转换成自然坐标系下的正方形标准单元。上式各量可插值基函数表示为

$$p = \sum_{e=1}^E \psi_n^{(e)} A_{ni}^{(e)} p_i, \quad h = \sum_{e=1}^E \psi_n^{(e)} A_{ni}^{(e)} h_i, \quad \varphi = \sum_{e=1}^E \psi_n^{(e)} A_{ni}^{(e)} \varphi, \quad \lambda = \sum_{e=1}^E \psi_n^{(e)} A_{ni}^{(e)} \lambda$$

式中, $A_{ni}^{(e)}$ 为第 e 个单元的布尔矩阵, $n = 1, 2, \dots, 8, i = 1, 2, \dots, N, N$ 为整个解区域的节点总数; $\psi_n^{(e)}$ 为第 e 个单元内第 n 个节点的形状函数^[3]; ζ, η 为自然坐标, h_i, φ, λ 分别为第 i 个节点值。

记 $P^e = (p_1, p_2, \dots, p_8)^T, H^e = (h_1, h_2, \dots, h_8)^T, \Psi = (\psi_1, \psi_2, \dots, \psi_8)^T,$

$\Psi_1 = \left(\frac{\partial \psi_1}{\partial \varphi}, \frac{\partial \psi_2}{\partial \varphi}, \dots, \frac{\partial \psi_8}{\partial \varphi} \right)^T, \Psi_2 = \left(\frac{\partial \psi_1}{\partial \lambda}, \frac{\partial \psi_2}{\partial \lambda}, \dots, \frac{\partial \psi_8}{\partial \lambda} \right)^T$ 。将上述关系式代入式(4), 得

$$\iint_{\epsilon} \Psi^T H^e)^3 (\Psi_1 \Psi_1^T + \alpha^2 \Psi_2 \Psi_2^T) \|J\| d\zeta d\eta A_{ni}^{(e)} P^e = 3\epsilon \iint_{\epsilon} \Psi \|J\| \sin \varphi d\zeta d\eta A_{ni}^{(e)}$$

$$\text{或} \quad K^e A_{ni}^{(e)} A_{nj}^{(e)} P^e = 3\epsilon W^e A_{ni}^{(e)} \quad (5)$$

式中, $|J| = \sum_{n=1}^8 \frac{\partial \psi_n}{\partial \zeta} \varphi_n \sum_{n=1}^8 \frac{\partial \psi_n}{\partial \eta} \lambda_n - \sum_{n=1}^8 \frac{\partial \psi_n}{\partial \eta} \varphi_n \sum_{n=1}^8 \frac{\partial \psi_n}{\partial \zeta} \lambda_n$ 为 Jacobi 行列式。

逐单元积分后迭加, 形成有限元线性代数方程组

$$P = 3\epsilon K^{-1} W \quad (6)$$

式中, $K = \sum_{e=1}^E K^e A_{ni}^{(e)} A_{nj}^{(e)}, W = \sum_{e=1}^E W^e A_{ni}^{(e)}, n = 1, 2, \dots, 8, i, j = 1, 2, \dots, N$

2 刚度、阻尼系数的解

无量纲动态 Reynolds 方程有限元方程为

$$\iint h^3 \text{Rey} \left\{ \frac{\partial \varphi}{\partial x_r} \right\} d\varphi d\lambda = 3 \iint \left[\frac{\partial \Psi}{\partial \varphi} - h^2 \text{Rey}(p) \right] \sin \varphi d\varphi d\lambda \quad (7)$$

$$\iint h^3 \text{Rey} \left\{ \frac{\partial \varphi}{\partial y_r} \right\} d\varphi d\lambda = 3 \iint \left[\frac{\partial \Psi}{\partial \varphi} - h^2 \text{Rey}(p) \right] \cos \varphi d\varphi d\lambda \quad (8)$$

$$\iint h^3 \text{Rey} \left\{ \frac{\partial \varphi}{\partial u_r} \right\} d\varphi d\lambda = -6 \iint \Psi \sin \varphi d\varphi d\lambda, \quad \iint h^3 \text{Rey} \left\{ \frac{\partial \varphi}{\partial v_r} \right\} d\varphi d\lambda = -6 \iint \Psi \cos \varphi d\varphi d\lambda \quad (9)$$

式中, x_r, y_r, u_r, v_r 分别为轴颈中心坐标 X, Y 方向的位移扰动和速度扰动, $\text{Rey}(\cdot) = \frac{\partial \cdot}{\partial \varphi} \frac{\partial \Psi}{\partial \varphi}$

+ $\alpha^2 \frac{\partial \cdot}{\partial \lambda} \frac{\partial \Psi}{\partial \lambda}$, $\varphi, \vartheta, \theta$ 为偏位角, 见图 1。

以式(7)为例, 参照式(6)的推导过程, 可得刚度系数 $K_{xx}, K_{xy}, K_{yx}, K_{yy}$ 表达式

$$\begin{aligned} K_{xx} &= 6W_s^T K^{-1} W_1 - 6W_s^T K^{-1} K_1 P, & K_{xy} &= 6W_c^T K^{-1} W_1 - 6W_c^T K^{-1} K_1 P \\ K_{yx} &= 6W_s^T K^{-1} W_2 - 6W_s^T K^{-1} K_2 P, & K_{yy} &= 6W_c^T K^{-1} W_2 - 6W_c^T K^{-1} K_2 P \end{aligned} \quad (10)$$

$$W_s = \sum_{e=1}^E \iint \Psi \|J\| \sin \varphi d\zeta d\eta A_{ni}^{(e)}, \quad W_c = \sum_{e=1}^E \iint \Psi \|J\| \cos \varphi d\zeta d\eta A_{ni}^{(e)}$$

$$W_1 = \sum_{e=1}^E \iint \Psi_1 \|J\| \sin \varphi d\zeta d\eta A_{ni}^{(e)}$$

$$K_1 = \sum_{e=1}^E \iint (\Psi^T H^e)^3 (\Psi_1 \Psi_1^T + \alpha^2 \Psi_2 \Psi_2^T) \|J\| \sin \varphi d\zeta d\eta A_{ni}^{(e)} A_{nj}^{(e)}$$

$$W_2 = \sum_{e=1}^E \iint \Psi_1 \|J\| \cos \varphi d\zeta d\eta A_{ni}^{(e)}$$

$$K_2 = \sum_{e=1}^E \iint (\Psi^T H^e)^2 (\Psi_1 \Psi_1^T + \alpha^2 \Psi_2 \Psi_2^T) \|J\| \cos \phi \, d\Omega \, \eta_{ni}^{(e)} A_{nj}^{(e)}$$

记 $P_s = K^{-1} W_s$, $P_c = K^{-1} W_c$, $P_1 = K_1 P$, $P_2 = K_2 P$ (11)

则 $P_c = \text{ctg} \theta P_s - p / (3 \epsilon \sin \theta)$ 。于是式(11)变为

$$\left. \begin{aligned} K_{xx} &= 6W_1^T P_s - 6P_1^T P_s \quad , \quad K_{xy} = 6W_1^T P_c - 6P_1^T P_c \\ K_{yx} &= 6W_2^T P_s - 6P_2^T P_s \quad , \quad K_{yy} = 6W_2^T P_c - 6P_2^T P_c \end{aligned} \right\} \quad (12)$$

参照式(12)的推导过程,由式(9)可得阻尼系数 D_{xx} 、 D_{xy} 、 D_{yx} 、 D_{yy} 的表达式

$$D_{xx} = -12W_s^T P_s, D_{xy} = -12W_c^T P_s \quad , \quad D_{yx} = -12W_s^T P_c, D_{yy} = -12W_c^T P_c \quad (13)$$

求出列阵 W_s 、 W_c 及 P_s , 刚度、阻尼系数便可得到, 只需求解一次 Reynolds 方程。

用该有限元法对椭圆轴承动特性系数进行了计算。见表 1($\alpha=1.25, \delta=0.5$, 进油口张角 30°)。在表 2 中给出了用差分法计算同种轴承的动特性系数。 δ 为预负荷系数。

表 1 椭圆轴承动特性表

ϵ	θ	K_{xx}	K_{xy}	K_{yx}	K_{yy}	D_{xx}	D_{xy}	D_{yx}	D_{yy}
0.10	79.38	3.213	-17.318	7.293	43.872	7.593	-13.181	-13.182	48.570
0.15	79.09	4.052	-17.420	10.002	47.106	8.020	-11.933	-11.933	54.237
0.20	77.43	5.119	-16.841	13.369	55.254	8.770	-10.146	-10.146	61.852
0.25	76.15	6.984	-15.276	18.947	73.865	9.895	-8.024	-8.022	72.640

表 2 椭圆轴承动特性表^[4]

ϵ	θ	K_{xx}	K_{xy}	K_{yx}	K_{yy}	D_{xx}	D_{xy}	D_{yx}	D_{yy}
0.10	79.51	3.487	-17.49	7.590	44.12	7.709	-13.87	-13.73	49.97
0.15	79.17	4.248	-17.52	10.599	47.82	8.320	-12.62	-12.59	56.98
0.20	78.07	5.440	-17.05	13.746	56.51	9.045	-10.78	-10.80	62.93
0.25	76.20	7.128	-15.38	20.193	74.19	10.06	-8.35	-8.48	73.99

用这种有限元法计算轴承动特性系数, 由于省去了几次 Reynolds 方程的求解, 因此比其他有限元法节省约 4~5 倍计算时间。由表 1 和表 2 可见, 用这种有限元法与差分法计算出来的同种轴承动特性系数基本吻合, 相对误差在 5% 以内, 是可信的。

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new trainer engine. This study shows that due to short average flight time, a great difference exists between the mission duty cycles of the trainer engine and the design requirements of Engine Structural Integrity Standard (MIL-STD-1783).

Key words Training aircraft Aircraft engines Loading spectrum

A FINITE ELEMENT METHOD FOR COMPUTING DYNAMIC COEFFICIENT OF HYDRODYNAMIC JOURNAL BEARINGS

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ABSTRACT A finite element method is presented for computing the oil-film dynamic coefficients of hydrodynamic journal bearings. Once the integral equations of the four functions, $\hat{q}_p/\hat{\alpha}_{xc}$, $\hat{q}_p/\hat{\alpha}_{yc}$, $\hat{q}_p/\hat{\alpha}_{xc}$, $\hat{q}_p/\hat{\alpha}_{yc}$, are gotten, their matrix expressions can be obtained immediately. Integrating them over the film domain, we can get the eight oil-film dynamic coefficients. Then it is possible to solve the Reynolds equation only once.

Key words Matrices Hydrodynamic journal bearings Finite element methods

AN INVESTIGATION ON VIBRATION MONITORING TECHNIQUE FOR A TURBOSHAFT ENGINE

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ABSTRACT The Vibration characteristics of the turboshaft engine on ground and in flight are considered. According to the distribution of the vibration frequency spectrum, a method is presented to display and record vibration velocity value of the turboshaft engine on ground or in flight. A vibration monitor made of INTEL 51 series 8031 single chip microcomputer has eight channels. Six channels are employed to record vibration signals and the other two channels to measure engine rotating speeds. The system operation performance has been proved by a large amount of measurements on fields and airborne measurements.

Key words Turboshaft engines Vibration Monitoring

A STUDY ON MOTION BEHAVIOURS OF PARTICLES THROUGH TURBINE CASCADE

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ABSTRACT A high speed camera is used to photograph the motion process of particles through turbine cascade. The effects of particle size and initial position on the particle trajectories and velocities are analyzed. The particle trajectories and velocities are also computed with the Lagrangian method under conditions of flow with and without boundary layer. Comparison between computational and experimental results shows that it is necessary to consider the effect of the boundary layer for the smaller particles, but it is unnecessary for the larger particles when the Lagrangian method is used to compute the motion behaviours of particles.

Key words Plane cascades Solid particles High-speed photography