

轮盘低周疲劳寿命的可靠性研究

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【摘要】 利用 R. W. Landgraf 的低周疲劳寿命公式, 建立累积损伤的低周疲劳寿命的失效方程, 以此方程为基础利用 H-L 方法求其可靠度。推导了在非线性的弹塑性状况下的随机有限元法, 从而使低周疲劳寿命可靠性得以计算。

主题词: 疲劳寿命 随机有限元 可靠度

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1 低周疲劳失效方程的建立

在高应变低周疲劳范围内, 弹性应变分量及塑性应变分量与循环寿命 N 的关系, 在对数坐标内均呈线性变化, 即

$$\Delta\epsilon_e/2 = \sigma'_f (2N)^{b/E}, \quad \Delta\epsilon_p/2 = \epsilon'_f E (2N)^c \quad (1)$$

式中 $\Delta\epsilon_e$ 、 $\Delta\epsilon_p$ 分别为弹性应变分量及塑性应变分量; σ'_f 为疲劳强度系数, b 为疲劳强度指数, ϵ'_f 为疲劳延性系数, c 为疲劳延性指数。

为计算累积损伤寿命方便, 采用 R. W. Landgraf 方法计算低周疲劳寿命, 这个公式主要考虑损伤是由 $\Delta\epsilon_e$ 与 $\Delta\epsilon_p$ 的比值来控制, 所以由上二式的比值, 并且如计入平均应力 σ_m 的影响, 修正后的损伤公式为:

$$\frac{1}{N} = 2 \left[\frac{\sigma'_f}{E \epsilon'_f} \cdot \frac{\Delta\epsilon_p}{\Delta\epsilon_e} \cdot \frac{\sigma'_f}{\sigma'_f - \sigma_m} \right]^{\frac{1}{b-c}} \quad (2)$$

再运用 Miner 线性累积损伤公式 $\sum_{i=1}^k \frac{n_i}{N_i} = a$, (n_i 为第 i 个循环实际循环数), 得出

$$\sum_{i=1}^k \left\{ 2n_i \left[\frac{\sigma'_f}{E \epsilon'_f} \cdot \frac{\sigma'_f}{\sigma'_f - \sigma_m} \right]^{\frac{1}{b-c}} \cdot \left(\frac{\Delta\epsilon_p}{\Delta\epsilon_e} \right)^{\frac{1}{b-c}} \right\} \leq a \quad (3)$$

整理上式则得用应变表示的失效方程:

$$g = a - \sum \left\{ 2n_i \left[\frac{\sigma'_f}{E \epsilon'_f} \cdot \frac{\sigma'_f}{\sigma'_f - \sigma_m} \right]^{\frac{1}{b-c}} \cdot \left(\frac{\Delta\epsilon_p}{\Delta\epsilon_e} \right)^{\frac{1}{b-c}} \right\} \quad (4)$$

利用应力-应变的关系:

$$\text{弹性时,} \quad \Delta\epsilon_e/2 = \Delta\sigma/2E = \sigma_a/E \quad (5)$$

$$\text{塑性时,} \quad \Delta\epsilon_p/2 = (\Delta\sigma/2K)^{\frac{1}{n}} = (\sigma_a/K)^{\frac{1}{n}} \quad (6)$$

σ_a 为应力幅值, 由(1)可推导材料系数之间关系得: 循环强度系数 $K \triangleq \sigma_f' / \epsilon_f'$; 循环应变硬化指数 $n \triangleq b/c$ 。

将(5)、(6)式代入(4)式并整理, 则得失效方程的应力表达式, 也就是计算可靠度所需之失效方程为:

$$g = a - \sum \left\{ 2n_i \left[\frac{\sigma_f'}{\sigma_f' - \sigma_m} \right]^{\frac{1}{b-c}} \cdot \left(\frac{\sigma_f'}{\sigma_a} \right)^{\frac{1}{b}} \right\}, \quad g = g(a, \sigma_f', b, c, \rho, \omega)$$

计算中取上式中的函数, $a, \sigma_f', b, c, \rho, \omega$ 为随机量, 计算轮盘疲劳寿命的可靠度。当 $g \leq 0$, 则失效; $g > 0$, 则安全。

所取载荷谱为三角波, 只考虑一种工作循环时, $a = 1$, 此时失效方程简化为:

$$g = 1 - 2n_i \left\{ \left(\frac{\sigma_f'}{\sigma_f' - \sigma_m} \right)^{\frac{1}{b-c}} \cdot \left(\frac{\sigma_f'}{\sigma_a} \right)^{\frac{1}{b}} \right\} \quad (7)$$

2 非线性随机有限元的计算过程及可靠度的推导

由(7)式在设计点按 Taylor 级数展开, 取线性项 $E[g]^* = g^* + \sum_k (\mu_{X_k} - X_k^*) \left(\frac{\partial g}{\partial X_k} \right)^*$ (* 下标表示代入各随机量在设计点的值。 X_k 为第 k 个随机量)。假定各随机量相互独立则其方差为: $V[g] = \sum_k \left(\frac{\partial g}{\partial X_k} \right)^2 V_{ar}[X_k]$, 可靠性指标为: $\beta = E[g] / (V[g])^{1/2}$ 。计算时, 仍按均值为初始值, 按下式迭代,

$$\alpha = \frac{\left[\frac{\partial g}{\partial X} (V_{ar}[X])^{1/2} \right]}{\left[\sum_k \left\{ \left(\frac{\partial g}{\partial X_k} \right)^2 (V_{ar}[X_k]) \right\} \right]^{1/2}}$$

第 j 次的迭代值:

$$x_j^* = \mu_j - \alpha \beta (V_{ar}[X])^{1/2}$$

可靠度则为: $R = \Phi(\beta)$ (详见参考文献[2])。

在计算 $\partial g / \partial X$ 中, 从(7)式可看出必会遇到求 $\partial \sigma_a / \partial X, \partial \sigma_m / \partial X$ 的值, 而 σ_a 的应力幅度是由有限元应力分析中得出的。 $\partial \sigma_a / \partial X$ 由随机有限元中求出^[3], 由于低周疲劳时的局部应力已进入弹塑性状态(非线性), 求位移用增量法, 采用变刚度法计算得出, 本文对 $\partial \sigma_a / \partial X$ 的计算也采用与求位移时的同样增量和同样刚度矩阵, 这样可计算非线性情况的可靠度, 其具体步骤如下:

$$[k] \{\delta\} = \{p\} \quad \{\delta\} = [k]^{-1} \{p\}$$

对于非线性情况 $[k]$ 不为常数, 则将载荷逐步加入, 每步修改 $[k]$ 值得增量位移。

$$\{\delta\} = \sum_{j=1}^n [k(\sigma_{j-1})]^{-1} \{\Delta p_j\} = \{\Delta \delta_1\} + \{\Delta \delta_2\} + \cdots + \{\Delta \delta_n\}, \quad \{\Delta \delta_j\} = [k(\sigma_{j-1})]^{-1} \{\Delta p_j\}$$

假定 $[k]$ 阵与随机量无关, 则得:

$$\frac{\partial \Delta \delta_j}{\partial X} = [k(\sigma_{j-1})]^{-1} \frac{\partial \Delta p_j}{\partial X}, \quad \Delta \sigma_j = [D(\sigma_j)]_{\varphi} [B] \{\Delta \delta_j\}, \quad \frac{\partial \Delta \sigma_j}{\partial X} = [D(\sigma_j)]_{\varphi} [B] \frac{\partial \Delta \delta_j}{\partial X}$$

式中 $[D(\sigma)]_{\varphi} = m[D]_e + (1-m)[D]_{\varphi}$, m 为过渡单元的加权系数。

$$\text{可得:} \quad \{\sigma_a\} = \sum \{\Delta \sigma_j\}, \quad \partial \sigma / \partial X = \sum_{j=1}^n \{\partial \Delta \sigma_j\} / \partial X$$

得出的非线性的应力对随机量偏导数, 代入 $\partial g / \partial X$ 的各式后, 即可得出 $E[g]$ 、 $V_{ar}[g]$ 及 β 值。具体的算式推导这里不再赘述。

在计算内径的应力时, 由于内径为自由面, 其径向应力 $\sigma_r = 0$, 按 Tresca 强度准则, 仅考虑周向应力分量为其合成应力的当量值。

3 算例

模型为离心力作用下空心等厚圆盘, 材料 TC11 内径为 $r_0 = 1\text{ cm}$, 外径 $r_b = 4\text{ cm}$, 厚度 $t = 1\text{ cm}$, $E = 1.21 \times 10^5\text{ MPa}$, $\nu = 0.33$, $\sigma_s = 816\text{ MPa}$, $\sigma_b = 1076\text{ MPa}$, $n = 47600\text{ RPM}$ 。

计算时首先对整个圆盘按弹性计算, 然后取分界面 $r_i = 3\text{ cm}$, 在 r_i 以内再进行弹塑性计算, 对于内径附近塑性区域小, 且界面以外形状复杂的盘例如离心式叶轮则更为有利, 对于复杂型面盘只进行一次弹性计算即可, 而进行非线性计算区域小且形状简单这可节省计算工作。界面的边界条件由线性状况下计算得出。与边界条件有关的随机量只有 2 个 ρ, ω 其它与材料性质有关的随机量与边界条件无关。各随机量分布均假设为正态分布, 非正态分布时可按 R-F (二参数法)。C-L (三参数法) 或 Y. T. Wu 的方法进行正态化后再按本方法进行计算。

表 1 基本随机量取值(离差系数均为 0.1)

随机变量	σ'_i (MPa)	ϵ'_i	b	c	ρ (kg/m ³)	ω (1/sec)
均值	1568	0.469	- 0.1	- 0.88	4.48×10^3	4974.188
方 差	156.8	0.0469	0.01	0.088	4.48×10^2	497.42

r_i 处的位移边界条件, 在 1、2、3 节点上, δ 都是 0.012 cm , $\partial\delta/\partial\rho$ 都等于 1.0325×10^2 , $\partial\delta/\partial\omega$ 都等于 1.9077×10^{-7} , 工作循环定为 5000 循环。

经过 8 次迭代, β 值收敛为 1.52632, 对应可靠度为 0.9365, 而用 Monte-Carlo 模拟时, 经过 100000 次计算, 可靠度为 0.9333。对比误差为 0.3%, 说明本计算方法及程序是可行的。如离差系数大于 0.2 时可用 Y. T. Wu 方法计算, 通过对本例计算表明效果较好。

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RELIABILITY ANALYSIS OF LOW-CYCLE FATIGUE LIFE FOR A DISC

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ABSTRACT

When a failure model with low-cycle fatigue is identified, the failure function for reliability analysis reduced as follows: As cumulative damage of varying-amplitude cycles is calculated with Miner's law, the total damage is considered to be a random value varying from 0.3 to 3, and follows logarithmic normal distribution. Furthermore, damage of low-cycle loads of each amplitude is evaluated according to the formula of flow-cycle fatigue life prediction proposed by R. W. Landgraf. Then, the failure function for reliability analysis is derived from the above formulas by transforming strain expression into stress expression. If a stress at certain point of a disk is influenced by a disturbance of some random variables, the stochastic finite element method is employed to find the variance of the stress distribution. When stresses in a disk are in elasto-plastic condition, nonlinear SFEM is required to obtain the variance of the stresses. Finally, a numerical example for a disk with equal thickness is presented and is verified by the Monte-Carlo simulation.

Key words Fatigue life Stochastic finite element Reliability

OPTIMIZATION OF TURBINE BLADE FIR-TREE ROOTS

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ABSTRACT

An optimization system for "fir-tree" root is developed for design layout of the "fir-tree" joint area in gas turbine so as to decrease the weight of aero engine and shorten the design cycle. This system is suitable to design the 2~5 pairs of teeth used in modern aero engines. After the optimization the smallest weight subject to the static strength constraints has been attained, the FEM is used to check the distribution of stresses, particularly the concentration stress, and the results are shown on the screen for making decision.

Key words Aircraft engine Fir-tree Blade root Optimization

COMMENT ON THE PROBLEM OF TURBULENCE

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ABSTRACT

This paper points out that Navier-Stokes equation is an essential equation for studying turbulent flows, it governs the motion of fluid flows. Reynolds equation represents coexistence of random and determinative phenomena in the turbulent flows. But, Burgers-KdV equation has no relation to mechanism of turbulence, so we can't regard it as a "new equation" for researching turbulent flows.

Key words Turbulence Navier-Stokes equations Reynolds