

循环对称接触结构的概率有限元应力分析

株洲航空动力机械研究所 杨治国* 尹泽勇

【摘要】 给出了循环对称接触表面上相应节点的循环对称接触关系及其关于随机变量的导数表达式,建立了考虑材料物性和载荷随机性的循环对称接触结构基本段概率有限元应力分析法。还给出了采用一次二阶矩 Hasofer-Lind 方法的相应结构强度可靠性分析公式。关于某发动机离心叶轮结构强度可靠性分析的实例表明了本方法的有效性与实用性。

主题词: 对称体 概率方法 有限元分析 结构可靠性
分类号: O316

1 循环对称接触关系及其关于随机变量的导数、结构概率有限元列式

在图 1 所示循环对称接触问题示例中,对基本段 I 的一对 5 自由度壳元类循环对称接触节点 i 和 j ,在不考虑摩擦力的情况下,其循环对称接触关系为:

循环对称一体接触

$$\{u\}_i = [t_1]\{u\}_j + [t_2][\delta_{ij} \ 0 \ 0 \ 0 \ 0]^T$$

$$\{f\}_i = -[t_1]\{f\}_j$$

$$\partial\{u\}_i/\partial x_k = [t_1]\partial\{u\}_j/\partial x_k$$

$$\partial\{f\}_i/\partial x_k = -[t_1]\partial\{f\}_j/\partial x_k \quad (k = 1, \cdots, m)$$

这里 $\{u\}_i$, $\{u\}_j$ 及 $\{f\}_i$ 和 $\{f\}_j$ 分别为节点 i 及节点 j 的位移向量和节点接触力向量; $[t_1]$ 和 $[t_2]$ 为两变换矩阵(详见文献[1]); $x_k (k = 1, \cdots, m)$ 为一组基本随机变量。

循环对称滑动接触

$$[\Theta_1]\{u\}_i = [\Theta_1][t_1]\{u\}_j + \{\delta_{ij} 0\}^T$$

$$[\Theta_1]\{f\}_i = -[\Theta_1][t_1]\{f\}_j, \quad [\Theta_2]\{f\}_i = -[\Theta_2][t_1]\{f\}_j = \{0\}$$

$$[\Theta_1]\partial\{u\}_i/\partial x_k = -[\Theta_1][t_1]\partial\{u\}_j/\partial x_k \quad (k = 1, \cdots, m)$$

$$[\Theta_1]\partial\{f\}_i/\partial x_k = -[\Theta_1]\partial\{f\}_j/\partial x_k$$

$$[\Theta_2]\partial\{f\}_i/\partial x_k = [\Theta_2][t_1]\partial\{f\}_j/\partial x_k = \{0\}$$

$$(k = 1, \cdots, m)$$

式中 $[\Theta_1]$ 和 $[\Theta_2]$ 是矩阵 $[t_2]$ 的子矩阵,详见文献[1]。

循环对称开式接触

$$\{f\}_i = \{f\}_j = \{0\}, \quad \partial\{f\}_i/\partial x_k = \partial\{f\}_j/\partial x_k = \{0\} \quad (k = 1, \cdots, m)$$

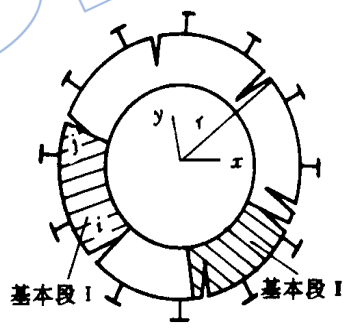


图 1 循环对称接触问题示例

2 可靠度计算

对各向同性材料, 根据 M isess 屈服条件, 临界函数 g 可定义为: $g= 1- \sigma_e/\sigma_{0.2}$, 其中 $g< 0$ 为失效, $g> 0$ 为安全。该临界函数 g 的期望值、方差分别为:

$$E[g] = 1 - \frac{\sigma_e^*}{\sigma_{0.2}^*} - \sum_{k=1}^m (\mu_{x_k} - x_k^*) \left[\frac{\partial \sigma_e/\sigma_{0.2}}{\partial x_k} \right]^*$$
$$\text{Var}[g] = \sum_{k=1}^m \sum_{l=1}^m \left[\frac{\partial \sigma_e/\sigma_{0.2}}{\partial x_k} \right]^* \left[\frac{\partial \sigma_e/\sigma_{0.2}}{\partial x_l} \right]^* \text{Cov}[x_k, x_l]$$

μ_{x_k} 为随机变量 x_k 的均值, x_k^* 为破坏点处随机变量 x_k 的值; σ_e 为 M isess 当量应力; $\text{Cov}[x_k, x_l]$ 为 x_k 与 x_l 协方差。

相应的可靠度指标 β 、破坏概率 P_f 及可靠度 R_e 分别为:

$$\beta = E[g]/(\text{Var}[g])^{1/2} \quad , \quad P_f = \Phi(-\beta)$$
$$R_e = 1 - P_f = \Phi(\beta)$$

式中 $\Phi()$ 表示标准正态累积分布函数。

值得指出, 计算开始并不知道破坏点, 为此采用 Ha-sofer- Lind 逼近法^[3]。

3 应用实例

图 2 所示为仅承受离心力的某机离心叶轮有限元计算模型。计算时取转速、材料密度及材料屈服极限作为基本随机变量。为了对比, 采用二种方法, 计算结果列于表 1。一种是本文所述方法, 计算结果记为 A; 另一种为 Monte- Carlo 法, 计算结果记为 B。

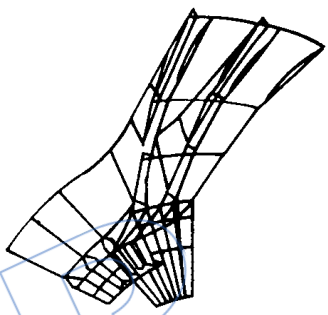


图 2 有限元计算模型

表 1 计算结果

可靠度 R_e		破坏概率 P_f		相对误差(%)
A	B	A	B	$(R_{eA} - R_{eB})/R_{eB}$
0.996207	0.996100	0.003793	0.003900	0.0107

由表 1 可知, 本文方法所得结果与 Monte- Carlo 方法相当接近。

参 考 文 献

1 Yin Zeyong and Nie Jingxu.The Finite Element Analysis for Cyclic Symmetric Contact Problems.In Applied Mechanics, Edited by Zheng Zhemin, International Academic Publisher, Pergamon Press, 1989, 3

2 杨治国, 尹泽勇. 循环对称接触结构的概率有限元应力分析. 第七届发动机结构强度振动学术会议论文集, 1994

3 Abraham M Hasofer, Niels C Lind. Ex act and Invariant Second- Moment Code Format. ASCE Journal of the Eng ineering Mechanics Division, 1974, 100(EML) 111- 121

AEROENGINE LIFE MONITORING AND REAL- TIME DATA PROCESSING

He Jinglan, Hong Jie, Li Qihan

(4th Dept. Beijing University of Aeronautics and Astronautics, Beijing 100083)

ABSTRACT

Aeroengine load and life monitoring technology play very important role in monitoring and assessing operating conditions, life consumption and remaining life of aeroengine. Aeroengine load can improve the safety and reliability of an engine in high performance operation, and reduce its direct operation cost. The on-board life monitoring system requires the real-time analysis method of flight data processing and life consumption evaluation. Three point method, shortest sail-route method and rain-flow method are used to conduct real-time data condensing processing, including peak valley values inspection, un-effective magnitude value detecting and LCF counting for each flight. The information obtained by the on-board life monitoring system is very useful for life track on the data base of the engine parts.

Key Words Aircraft engines Afterburners Transient process Control problems

PROBABILISTIC FINITE ELEMENT STRESS ANALYSIS FOR CYCLIC SYMMETRIC CONTACT STRUCTURE

Yang Zhiguo and Ying Zeyong

(Zhuzhou Aircraft Powerplant Research Institute, Zhuzhou 412002)

ABSTRACT

The cyclic symmetric contact relation of corresponding node pairs on cyclic symmetric contact surfaces is given and its derivative expression with respect to random variables is deduced. A probabilistic finite element stress analysis technique is developed for the basic segment of the cyclic symmetric contact structure including random material property and load. A structure strength reliability analysis formula is also established with the first-order second moment method proposed by Hasofer and Lind. Finally the structure strength reliability analysis for a centrifugal impeller of an engine is carried out by using the method presented. The results show the effectiveness and practicability of the present method not only for the cyclic symmetric contact structure, but also for other non-cyclic symmetric contact structures.

Key Words Symmetrical bodies Probabilistic methods Finit element analysis
Structural reliability

A STRUCTURE LIFE EVALUATION MODEL AND ITS APPLICATION TO DYNAMIC COMPONENTS OF AIRCRAFT POWERPLANT

Xiong Junjiang, Huang Xinyu, Gao Zhengtong

(5th Dept. Beijing University of Aeronautics and Astronautics, Beijing 10083)

Gan Weimin, Sun Rulin, Sun Shuling

(Chinese Helicopter Research Institute, Jingdezhen 333001)

ABSTRACT

The life of a structure is evaluated with its safe-life design, and its safety is ensured with the damage tolerant design, so the safe-life design and the damage

在此值得指出,只要随机变量的分布特性是针对整个结构而不是针对单独一个基本段来确定的,这里所建立的循环对称接触关系及其对随机变量的导数公式就是有效的。

对于有一对循环对称接触节点 i 与 j 的基本段 I (图 1 所示),利用上面建立的节点对 i 与 j 处的循环对称接触关系及其关于随机变量的导数表达式,引入文献[1]中的 4 个乘子矩阵 $[C]$ 、 $[C_1]$ 、 $[C_2]$ 及 $[C_3]$,可得修正后基本段概率有限元方程的统一表达式:

$$\begin{aligned} & [K'] \cdot [\{u\}_1 \cdots \{\bar{u}\}_i \cdots \{\bar{u}\}_j \cdots \{u\}_n]^T \\ &= [\{p_1\} \cdots [C_2][t_2]^T \{p_i\} \cdots [C][t_2]^T \{p_i\} + [t_2]^T [t_1] \{p_j\} \cdots \{p_n\}]^T \\ &- \delta_{ij} [[k_{ii}] \cdots [C_2][t_2]^T [k_{ii}] \cdots [C_1][t_2]^T [k_{ii}] + [t_2]^T [t_1] [k_{jj}] \cdots [k_{ni}]]^T [t_2][C_3] \end{aligned} \quad (1)$$

$$[K'] \cdot \begin{bmatrix} \partial \{u\}_1 / \partial x_k \\ \vdots \\ \partial \{\bar{u}\}_i / \partial x_k \\ \vdots \\ \partial \{\bar{u}\}_j / \partial x_k \\ \vdots \\ \partial \{u\}_n / \partial x_k \end{bmatrix} = \begin{bmatrix} \{p_1\}_{x_k} - \{p_1\}_{x_k}^f \\ \vdots \\ [C_2][t_2]^T (\{p_i\}_{x_k} - \{p_i\}_{x_k}^f) \\ \vdots \\ [C_2][t_2]^T (\{p_i\}_{x_k} - \{p_i\}_{x_k}^f) + [t_2]^T [t_1] (\{p_j\}_{x_k} - \{p_j\}_{x_k}^f) \\ \vdots \\ \{p_n\}_{x_k} - \{p_n\}_{x_k}^f \end{bmatrix} \quad (k = 1, \cdots, m) \quad (2)$$

式中 $[K']$ 为修正的刚度阵,详见文献[2]; $[k_{ij}]$ 等是原刚度阵子阵; $\{p\}_i$ 是节点外载向量;且:

$$\begin{aligned} \{u\}_i &= [t_2] \{\bar{u}\}, \quad \{u\}_j = [t_1]^T [t_2] \{\bar{u}\}_j, \quad \partial \{p_i\} / \partial x_k = \{p_i\}_{x_k} \\ (\partial [K] / \partial x_k) \{u\} &= [\{p_1\}_{x_k}^f \{p_2\}_{x_k}^f \cdots \{p_i\}_{x_k}^f \cdots \{p_j\}_{x_k}^f \cdots \{p_n\}_{x_k}^f]^T \\ \partial \{u\}_i / \partial x_k &= [t_2] \partial \{\bar{u}\}_i / \partial x_k, \quad \partial \{u\}_j / \partial x_k = [t_1]^T [t_2] \partial \{\bar{u}\}_j / \partial x_k \end{aligned}$$

这里的 $\{u\}$ 是基本段的节点位移向量。有多对循环对称接触节点的公式与上述公式类同。

有限元分析中,单元刚度及单元刚度导数的一般表达式为:

$$[K]^e = \int [B]^T [D] [B] dv, \quad \frac{\partial [K]}{\partial x_k} = \int [B]^T \frac{\partial [D]}{\partial x_k} [B] dv \quad (k = 1, \cdots, m) \quad (3)$$

这里 $[D]$ 为单元材料弹性矩阵, $[B]$ 为应变位移矩阵。

$$\{R\}^e = \int [N]^T \{b\} dv + \int [N]^T \{t\} ds + \int [B]^T [D] \{\epsilon_0\} ds \quad (4)$$

单元载荷向量及其导数的一般表达式为:

$$\begin{aligned} \frac{\partial \{R\}}{\partial x_k} = & \int [N]^T \frac{\partial \{b\}}{\partial x_k} dv + \int [N]^T \frac{\partial \{t\}}{\partial x_k} ds + \int [B]^T \frac{\partial [D]}{\partial x_k} \{\epsilon_0\} dv + \int [B]^T [D] \frac{\partial \{\epsilon_0\}}{\partial x_k} dv \\ & (k = 1, \cdots, m) \end{aligned} \quad (5)$$

式中 $[N]$ 为形状函数矩阵; $\{b\}$ 和 $\{t\}$ 分别为一点处的体力向量和面力向量; $\{\epsilon_0\}$ 为初始热应变。

显然由式(3) - (5)即可得到式(1)与(2)中所需的 $\{p_i\}$ 、 $\{p_i\}_{x_k}$ 及 $\partial [K] / \partial x_k$ 。

元素内一点的应力向量 $\{\sigma\}$ 及其导数 $\partial \{\sigma\} / \partial x_k$ 的一般表达式为:

$$\{\sigma\} = [D] ([B] \{u\}^e - \{\epsilon_0\}) \quad (6)$$

$$\begin{aligned} \frac{\partial \{\sigma\}}{\partial x_k} = & \frac{\partial [D]}{\partial x_k} ([B] \{u\}^e - \{\epsilon_0\}) + [D] \left([B] \frac{\partial \{u\}}{\partial x_k} - \frac{\partial \{\epsilon_0\}}{\partial x_k} \right) \\ & (k = 1, \cdots, m) \end{aligned} \quad (7)$$