

三维对流项在 CVFEM 中的一种质量加权迎风方案

西北工业大学 王 旭* 严传俊

【摘要】 对三维对流项在四面体单元上建立了一种各结点控制体相一致的一种质量加权迎风方案,它既保留了质量加权方案对大的 Peclet 数及非结构式几何域的适用性,又简化了计算机程序的实现过程。

主题词: 有限元法 三维问题 加权平均值
分类号: O241.4

控制体有限元法(CVFEM)对三维问题的应用相对还不够成熟。文献[1]在四面体单元上建立的流向型指数迎风方案(FLO),对不规则单元或过大的 Peclet 数,会引起离散化方程的系数为负而引起解的振荡或发散。文献[2]在四面体单元上建立的质量加权迎风方案(MAW),虽然保证了离散化方程的系数为正,但对每个四面体单元结点所建立的不对称的子控制体对计算机程序的实现造成了一定的难度。本文给出四面体单元上各结点子控制体相对称的一种MAW方案,并采用L-U分解法加速其实现过程,用两个三维算例对其予以验证。

1 控制体及控制体积分方程

将计算域剖分为四面体单元,从每个单元的形心点分别向其四个面中点作连线,再将每一个面中点分别与其相应的三个边中点相连,所形成的四边形子控制面便对每一结点构成一个控制体 V 。根据守恒原理,在 V 上便有对流-扩散方程的积分形式

$$\int_V \mathbf{J} \cdot \mathbf{n} ds = \int_V S dV \quad (1)$$

其中 $\mathbf{J} = \rho \mathbf{v} \phi - \Gamma \nabla \phi$ 为对流-扩散通量, ∂V 为 V 的表面, $\mathbf{n}$ 为微元 ds 上的单位外法向量, ρ 为流体密度, Γ 为扩散系数, $\mathbf{v}$ 为流体流动速度。如图1所示,当结点 P 为网格域内点时, ∂V 在该单元上仅包括 $atos$ 、 $crot$ 及 $dsor$ 三个子控制面,否则,它在该单元上还将包括 $Pasd$ 、 $Pcta$ 及 $Pdrc$ 中的一个或几个。

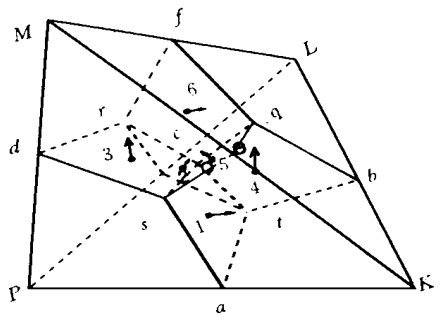


图 1 典型四面体单元及其子控制面上的单位法向量

2 本文建立的 MAW 方案

在图 1 中六个子控制面的形心点上按图示方向规定其上的单位法向量, 记其上的质量流量分别为:

$$\dot{m}_k = \int_{V_k} \rho \mathbf{v}_k \cdot \mathbf{n}_k ds = \left(\rho \mathbf{v}_k \cdot \mathbf{n}_k \right) \delta V^k \quad (k = 1, \dots, 6)$$

δV^k 表示第 k 个子控制面及其面积, 则其上 6 个子控制面上的对流项变量 ϕ 可分别定义为:

$$\phi_1 = \begin{cases} f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(\dot{m}_1, \dot{m}_2, \dot{m}_3) \quad \dot{m}_1 > 0 \\ f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(-\dot{m}_1, \dot{m}_4, \dot{m}_5) \quad \dot{m}_1 < 0 \end{cases} \quad (2)$$

$$\phi_2 = \begin{cases} f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(\dot{m}_2, \dot{m}_1, \dot{m}_3) \quad \dot{m}_2 > 0 \\ f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(-\dot{m}_2, -\dot{m}_4, -\dot{m}_6) \quad \dot{m}_2 < 0 \end{cases} \quad (3)$$

$$\phi_3 = \begin{cases} f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(\dot{m}_3, \dot{m}_1, \dot{m}_2) \quad \dot{m}_3 > 0 \\ f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(-\dot{m}_3, -\dot{m}_5, -\dot{m}_6) \quad \dot{m}_3 < 0 \end{cases} \quad (4)$$

$$\phi_4 = \begin{cases} f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(\dot{m}_4, -\dot{m}_1, \dot{m}_5) \quad \dot{m}_4 > 0 \\ f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(-\dot{m}_4, -\dot{m}_2, -\dot{m}_6) \quad \dot{m}_4 < 0 \end{cases} \quad (5)$$

$$\phi_5 = \begin{cases} f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(\dot{m}_5, -\dot{m}_1, \dot{m}_4) \quad \dot{m}_5 > 0 \\ f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(-\dot{m}_5, -\dot{m}_3, \dot{m}_6) \quad \dot{m}_5 < 0 \end{cases} \quad (6)$$

$$\phi_6 = \begin{cases} f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(\dot{m}_6, -\dot{m}_3, \dot{m}_5) \quad \dot{m}_6 > 0 \\ f\phi_1 + g\phi_1 + (1-f-g)\phi_1 & (f, g) = T(-\dot{m}_6, -\dot{m}_2, -\dot{m}_4) \quad \dot{m}_6 < 0 \end{cases} \quad (7)$$

假定 $\dot{m}_1 > 0$, 则加权因子 f, g 与 $\dot{m}_1, \dot{m}_2, \dot{m}_3$ 的对应关系为:

$$(f, g) = T(\dot{m}_1, \dot{m}_2, \dot{m}_3) = \begin{cases} (0, 0) & \dot{m}_2 > 0, \dot{m}_3 > 0 \\ (0, \min[-\dot{m}_3/(\dot{m}_1 + \dot{m}_2), 1]) & \dot{m}_2 > 0, \dot{m}_3 < 0 \\ (\min[-\dot{m}_2/(\dot{m}_1 + \dot{m}_3), 1], 0) & \dot{m}_2 < 0, \dot{m}_3 > 0 \\ (\dot{m}_2/(\dot{m}_2 + \dot{m}_3), \dot{m}_3/(\dot{m}_2 + \dot{m}_3)) & \dot{m}_2 < 0, \dot{m}_3 < 0 \text{ 且 } -(\dot{m}_2 + \dot{m}_3) \geq \dot{m}_1 \\ (-\dot{m}_2/\dot{m}_1, -\dot{m}_3/\dot{m}_1) & \dot{m}_2 < 0, \dot{m}_3 < 0 \text{ 且 } -(\dot{m}_2 + \dot{m}_3) < \dot{m}_1 \end{cases}$$

记 $\{\phi\} = (\phi_1, \dots, \phi_6)^T$, $\{\phi\} = (\phi_1, \dots, \phi_6)^T$, 式(2) - (7) 可改写成矩阵形式

$$[A]_{6 \times 6} \{\phi\} = [B]_{6 \times 4} \{\phi\} \quad (8)$$

$[A]_{6 \times 6}$ 是主对角线上元素全为 1 的矩阵, 无需选主元而直接采用 L-U 分解法对其求逆。从而式(8)可再改写为

$$\{\phi\} = [A]_{6 \times 6}^{-1} [B]_{6 \times 4} \{\phi\} = [C]_{6 \times 4} \{\phi\}$$

结合对扩散项的线性插值, 对网格域中的每一结点 P , 完成(1)左端在其所有相邻单元相关子控制面上的积分, 并完备式(1)右端的源项积分及组装, 即得结点 P 的离散化方程

$$a_P \phi = \sum a_{nb} \phi_{nb} + b_P \quad (9)$$

其中 nb 取遍 P 的所有相邻结点。

对式(9)可采用逐点、逐行、共轭梯度法或波前法求解^[3]。在采用逐点法或逐行法求解时, 松弛因子取为 0.7。

3 算例及结论

算例 1^[1]: 如图 2 所示, 设计算域为一单位立方体, $ABCD$ 为过计算域形心点的四边形平

面, ϕ 的边界值在 $ABCD$ 的下侧为 0, 上侧为 1, 在 $ABCD$ 的边线上为 0.5。在 $ABCD$ 与水平方向夹角 α 为 $45^\circ, 0^\circ, -45^\circ$ 时, 流体的流动速度分别为

$$\text{I: } u = 1/\sqrt{3}, \quad v = 1/\sqrt{3}, \quad w = 1/\sqrt{3}$$

$$\text{II: } u = 1/\sqrt{2}, \quad v = 1/\sqrt{2}, \quad w = 0$$

$$\text{III: } u = 1/\sqrt{3}, \quad v = 1/\sqrt{3}, \quad w = -1/\sqrt{3}$$

在 $S = 0, \Gamma \rightarrow 0$ 时, 这是一个纯对流问题, 计算域边界上 ϕ 的台阶变化值在对整个计算域的传输过程中应是保持不变的。在 $11 \times 11 \times 11$ 的均匀网格上, 分别按 FLO 及本文的 MAW 方案所得 $\overline{EG}$ 上的计算结果如图 3 所示。

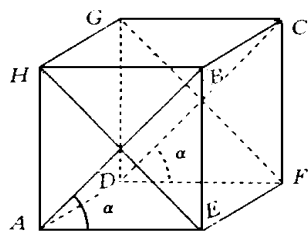


图 2 算例 1 的计算域

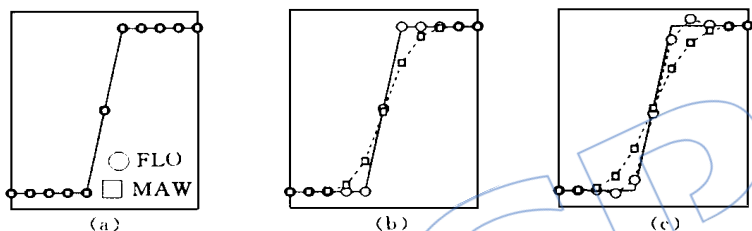


图 3 算例 1 在 $\overline{EG}$ 上的计算结果 (a. $\alpha = 45^\circ$; b. $\alpha = 0^\circ$; c. $\alpha = -45^\circ$)

算例 2^[1]: 在内半径 $r_1 (= 1)$, 外半径 $r_2 (= 2r_1)$ 的球壳内第一卦限部分取一固定立方体 R , 球壳内表面温度 T_1 , 外表面温度 $T_2 (T_2 < T_1)$ 。当球壳以角速度 $\omega = \omega \mathbf{i}$ 旋转时, R 内流体速度及无量纲温度场分别为:

$$\mathbf{u} = (\omega \mathbf{i}) \times (x \mathbf{i} + y \mathbf{j} + z \mathbf{k}) \quad (10)$$

$$\theta = 2r_1 / \sqrt{x^2 + y^2 + z^2} - 1 \quad (11)$$

将 R 剖分为 $11 \times 11 \times 11$ 的均匀网格, 边界条件按式 (11) 规定。图 4 给出了 $\rho = 1, Pe = 1/\Gamma$ 时本文 MAW 插值方案在 R 中心结点上的百分比误差:

$$\epsilon = 100 |\theta_{\text{解析解}} - \theta_{\text{数值解}}| / \theta_{\text{解析解}}$$

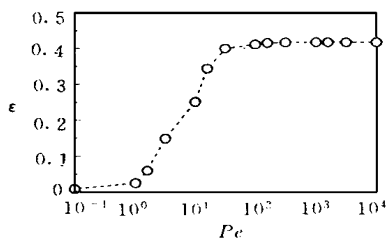


图 4 算例 2 在中心结点上的百分比误差

从上述算例可以看出, 虽然本文给出的 MAW 插值方案与 FLO 插值方案相比具有较强的假扩散, 但总体而言, 其精度仍是较高的。作者已将该方案用在了三维流动问题的计算之中, 且取得了较为满意的结果^[3]。

参 考 文 献

- 1 LeDain- Muir B, Baliga B R. Solution of Three- Dimensional Convection- Diffusion Problems Using Tetrahedral Elements and Flow- Oriented Upwind Interpolation Functions. Numer. Heat Transfer, 1986, 9: 143- 162
- 2 Saabas H J, Baliga B R. Co- Located Equal- Order Control- Volume Finite- Element Method for Multidimensional, Incompressible Fluid Flow-Part 1: Formulation. Numer. Heat Transfer, Part B, 1994, 26: 381- 407
- 3 王旭. 用于流动与传热问题的控制体有限元方法研究: [学位论文]. 西北工业大学, 1996

velocity distribution on the odd measuring plane is different from that on the even measuring plane. Channel vortex can be observed on the odd measuring plane, and horseshoe vortex is caught on the even measuring plane.

Key Words Spoilers Vortex Flow fields

A MAW UPWIND SCHEME FOR 3- D CONVECTION TERM

Wang Xu and Yan Chuanjun

(7th Dept. Northwestern Polytechnical University, Xi'an 710072)

ABSTRACT A new mass- weighted (MAW) upwind interpolation scheme, the control- volume finite- element method, is adopted for three dimensional convection term based on tetrahedral element, and its subcontrol volumes and surfaces on each element are symmetrical. This feature not only guarantees the positive coefficients in the discretization equation, but also is easy to be fulfilled in the computer program. The scheme has been applied successfully to some three dimensional convection- diffusion problems and some three dimensional viscous, incompressible fluid flow problems.

Key Words Finite element methods Weighted Three- dimensional probleme

SURVEY PAPERS

A SURVEY ON APPLICATION OF TBC FOR AEROENGINE HOT SECTIONS

Yang Xiaoguang, Gen Rui, Xiong Changbin

(4th Dept. Beijing University of Aeronautics and Astronautics, Beijing 100083)

ABSTRACT This paper gives a detailed survey about application of Thermal Barrier Coating (TBC) technology to aeroengines. First of all, thermal barrier coating system and coating technology are examined to identify the state of art of the coating technique, then characteristics of Plasma Spray Coating are compared with those of Electric Beam- Physic Vapor Deposition Coating. Meanwhile, the coating failure mechanism, coating interface strength and life prediction method are also surveyed. According to the thermal barrier coating researches at home and abroad, the authors offer some suggestions to promote the Chinese thermal barrier coating research and application.

Key Words Aircraft engines Heat insulation layer Ceramics

MECHANICAL BEHAVIORS OF TBC IN AEROENGINE HOT SECTION

Xiong Changbing and Yang Xiaoguang

(4th Dept. Beijing University of Aeronautics and Astronautics, Beijing 100083)

ABSTRACT A systematical study on the mechanical behaviors of Thermal Barrier Coating (TBC), the thermal compatibility of TBC with basic material of turbine blade, and the interface residual stress caused by various TBC materials. The change of the blade working temperature leads to crack, separation and scars between TBC and substrate, that results in TBC failure, so the blade cannot work normally. It is discovered from the experiments that addition of trace elements Zr- Y- Nb- AL in the thermal barrier coating as the