

运用低雷诺数非线性 $k-\epsilon$ 模型 研究湍流分离流动*

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【摘要】 给出了一个低雷诺数非线性 $k-\epsilon$ 湍流模型, 其中采用了一个非线性雷诺应力表达式并引入了两个考虑湍流近壁效应的修正函数。应用该模型和标准 $k-\epsilon$ 模型计算了两种不同膨胀比的后台阶分离流。与实测结果比较表明, 低雷诺数非线性模型结果较标准 $k-\epsilon$ 模型结果有明显改进, 它对流场的各特征量包括分离一再附长度、时均速度及各雷诺应力分量均给出了较好的预测。

主题词: 湍流特性 各向异性场 流动分离

分类号: O357.5

1 引言

由于湍流分离流动所包含的复杂流动形态及湍流各向异性, 现有大多数湍流模型不能对它给出合理的预测结果^[1]。作者发展了一个各向异性的 $k-\epsilon$ 模型^[2], 其特点是考虑了输运量扩散过程各向异性并采用了各向异性的雷诺应力表达式, 同时还引入了修正函数以考虑湍流的近壁效应。应用于壁面剪切流的结果表明该各向异性模型较其它模型有显著改进。作者基于该各向异性模型中的非线性雷诺应力表达式和近壁修正函数, 构成低雷诺数非线性 $k-\epsilon$ 模型, 并应用于计算几种不同膨胀比的后台阶分离流, 以期改进湍流分离流的计算并揭示其中各湍流量的变化规律。

2 低雷诺数非线性 $k-\epsilon$ 模型

基于文献[2]中各向异性模型的非线性雷诺应力表达式及近壁修正函数, 构成的低雷诺数非线性 $k-\epsilon$ 模型如下:

$$\frac{\partial k}{\partial t} + \frac{\partial (ku)}{\partial x} + \frac{\partial (kv)}{\partial y} = \frac{\partial}{\partial x} \left[\left(c_k f_1 \frac{k^2}{\epsilon} + \nu \right) \frac{\partial k}{\partial x} \right] + \frac{\partial}{\partial y} \left[\left(c_k f_1 \frac{k^2}{\epsilon} + \nu \right) \frac{\partial k}{\partial y} \right] + p_k - \epsilon \quad (1)$$

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial (\epsilon u)}{\partial x} + \frac{\partial (\epsilon v)}{\partial y} = \frac{\partial}{\partial x} \left[\left(c_\epsilon f_1 \frac{k^2}{\epsilon} + \nu \right) \frac{\partial \epsilon}{\partial x} \right] + \frac{\partial}{\partial y} \left[\left(c_\epsilon f_1 \frac{k^2}{\epsilon} + \nu \right) \frac{\partial \epsilon}{\partial y} \right] + c_{\epsilon 1} p_k \frac{\epsilon}{k} - c_{\epsilon 2} f_2 \frac{\epsilon^2}{k} \quad (2)$$

$$p_k = - \bar{u}^2 \frac{\partial u}{\partial x} - \bar{v}^2 \frac{\partial v}{\partial y} - \overline{uv} \left\{ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\} \quad (3)$$

$$\bar{u}^2 = \frac{2}{3} k - 2c_{\mu} f_1 \frac{k^2}{\epsilon} \frac{\partial u}{\partial x} - \frac{1}{3} c_{\mu} f_1 \frac{k^3}{\epsilon^2} \left[(c_1 - 2c_3) \left(\frac{\partial v}{\partial x} \right)^2 + (c_1 + c_2 + c_3) \left(\frac{\partial v}{\partial y} \right)^2 \right]$$

$$+ (c_3 - 2c_1) \left[\left(\frac{\partial u}{\partial y} \right)^2 - 2(c_1 + c_2 + c_3) \left[\left(\frac{\partial u}{\partial x} \right)^2 - c_2 \frac{\partial v}{\partial x} \frac{\partial u}{\partial y} \right] \right] \quad (4)$$

$$\bar{v}^2 = \frac{2}{3}k - 2c_{\mu}f_1 \frac{k^2}{\epsilon} \frac{\partial}{\partial y} - \frac{1}{3}c_{\mu}f_1 \frac{k^3}{\epsilon^2} \left[(c_1 - 2c_3) \left[\left(\frac{\partial u}{\partial y} \right)^2 + (c_1 + c_2 + c_3) \left[\left(\frac{\partial u}{\partial x} \right)^2 \right. \right. \right. \\ \left. \left. \left. + (c_3 - 2c_1) \left[\left(\frac{\partial v}{\partial x} \right)^2 - 2(c_1 + c_2 + c_3) \left[\left(\frac{\partial v}{\partial y} \right)^2 - c_2 \frac{\partial v}{\partial x} \frac{\partial u}{\partial y} \right] \right] \right] \right] \right] \quad (5)$$

$$\overline{u'v'} = -c_{\mu}f_1 \frac{k^2}{\epsilon} \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right] + c_{\mu}f_1 \frac{k^3}{\epsilon^2} \left[\left[\frac{1}{2}c_2 + c_3 \right] \left[\frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} \right] \right. \\ \left. + \left[c_1 + \frac{1}{2}c_2 \right] \left[\frac{\partial u}{\partial x} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} \right] \right] \quad (6)$$

$$f_1 = \left[1 + \frac{3.45}{R_t} \right] \left[1 - \exp \left[- \frac{\gamma^+}{cd_1} \right] \right], \quad f_2 = \left\{ 1 - \frac{2}{9} \exp \left[- \left(\frac{R_t}{6} \right)^2 \right] \right\} \left[1 - \exp \left[- \frac{\gamma^+}{cd_2} \right] \right]^2 \quad (7)$$

$$R_t = k^2 / (\epsilon v), \quad \gamma^+ = \gamma u^* / \nu, \quad c_k = 0.09, \quad c_\epsilon = 0.069, \quad c_1 = 0.08 \\ c_2 = 0.045, \quad c_3 = -0.015, \quad c_\mu = 0.09, \quad c_{d1} = 1.4, \quad c_{d2} = 1.8 \quad (8)$$

上述方程中, x 和 y 分别是主流方向及其垂直方向的坐标, u 和 v 分别是 x 和 y 方向的速度, ν 是流体运动粘度, u^* 是壁面摩擦速度。 f_1 和 f_2 为考虑湍流近壁效应的修正函数, 在充分发展壁面剪切流中, $cd_1 = 70$ 和 $cd_2 = 5.0$; 对本文分离流动, 通过将计算结果与后文所述的实验结果比较, 得出较适宜的值为 $cd_1 = 50$ 和 $cd_2 = 4.4$ 。与标准 $k-\epsilon$ 模型不同, 上述模型中雷诺应力是流体变形速度的非线性表达式, 这在合理计算湍流各向异性时尤为重要^[2,3]。在模型常数 c_1 、 c_2 和 c_3 趋向零时, 上述模型转化为常规的低雷诺数 $k-\epsilon$ 模型。

3 结果及其分析

对流动的动量方程、连续方程及湍流模型方程所组成的控制方程组采用基于有限控制容积的 SIMPLER 法进行差分和求解, 其中对流项的差分采用幂函数方案。分别采用本文低雷诺数非线性模型(ANI 模型)和标准 $k-\epsilon$ 模型(STD 模型), 计算了膨胀比(本文定义为入口槽宽与台阶高度之比)为 8:1 和 2:1 的后台阶湍流分离流。入流边界取在台阶上游一个台阶高度位置, 各湍流量分布按充分发展槽流分布给出。对于 ANI 模型, 壁面边界条件为非滑移边界条件; 对于 STD 模型, 壁面边界条件采用壁面律函数。

3.1 膨胀比为 8:1 情况

此时台阶高度为 1.27 cm, 入口流动的马赫数为 0.128, 基于入口平均流 U_{in} 及台阶高度 H 计算的雷诺数为 31250。计算网格为 60×50 , 采用 ANI 模型时, 网格在近壁及回流一再附区被重点加密, 在 $\gamma^+ < 1.0$ 范围内布置了近 10 个网格点; 对 STD 模型, 网格则在两维方向上均匀分布。图 1~3 示出了相应的计算结果。部分图中还给出 Driver 等人的实验结果^[4]。

图 1 为计算的流线图, 根据壁面处垂直方向的速度导数为零的点判断, ANI 模型预测的无量纲回流一再附长度为 5.8, 该结果与实测的回流一再附长度(为 6.0)非常接近。台阶角部的角涡也明显可见。由 STD 模型给出的回流一再附长度为 5.2, 且没能预测出相应的角涡。由图可见, STD 模型预测的分离再附点附近出现物理上非真实的扭曲流线。类似的现象也在 Obi^[5] 和 Lasher^[1] 等人的研究中出现。Lasher 解释为由于近壁采用较密的网格所致, 但在本文中, ANI 模型的近壁网格比 STD 模型的近壁网格密得多, 其预测流线没有出现上述现象。可

见该非真实物理流线的出现除与网格布置有关外,还与具体湍流模型有关。

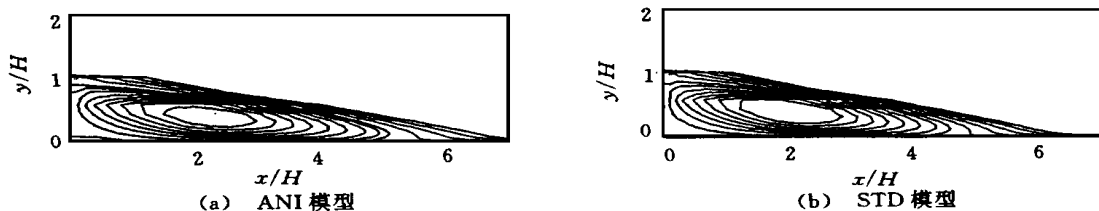


图 1 膨胀比为 8:1 的流线图

图 2(□文献[4]实验结果, —ANI 模型结果, ⋯STD 模型结果) 示出了在 $x/H = 6.0$ 处(实测回流一再附点)的预测时均速度及各雷诺应力分量与实测结果的比较。在该位置,由于 STD 模型预测的流场已离其预测再附点较远,因而该模型的时均速度与实测结果有较大差异; ANI 模型则给出了与实测值较一致的时均速度,最大回流速度为入口平均流速的 16%。ANI 模型预测的各雷诺应力分量最大值位置与实测结果一致, y 坐标方向雷诺正应力最大值和实测结果非常吻合,其余二应力最

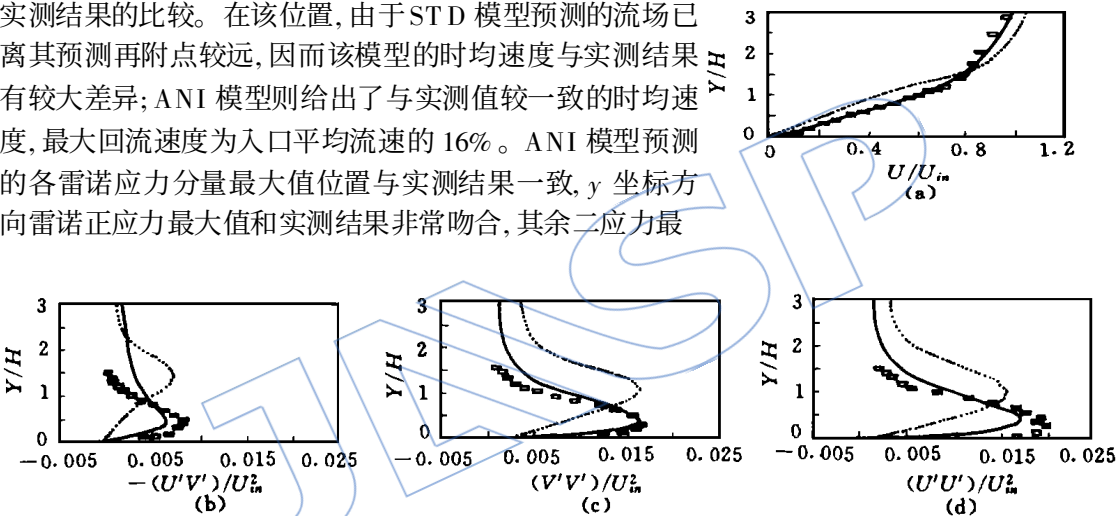


图 2 预测时均速度和各雷诺应力分量与相应实测结果的对比

大值则比实测结果小约 18%。这与 Sindir^[6]和 Lasher^[1]的雷诺应力模型预测结果相反。上述情况下, Sindir 和 Lasher 预测结果的误差分别为 35% 和 25% – 30%。由图可见,由 STD 模型给出的各雷诺应力分量最大值位置与实测结果相差较大。比较结果还表明,在回流区段($x/H = 4.0$),由 ANI 模型预测的各雷诺应力分量最大值位置仍与实测一致,与图 2 情况不同的是,此时除 y 坐标方向正应力最大值略大于实测值外,其余二应力最大值与实测值一致。

3.2 膨胀比为 2:1 情况

此时台阶高度为 3.81 cm,以入口平均流 U_{in} 及台阶高度 H 计算的雷诺数为 72500。计算网格为 45×40 , ANI 模型的网格布置同样在分离流区及近壁区重点加密,并保证在 $y^+ < 0.6$ 范围内至少有 3 个网格点, STD 模型网格在两维方向均布。

此时由 ANI 模型及 STD 模型预测的回流一再附长度分别为 6.8 和 6.6,相应的最大回流速度分别为入口平均流速的 22.3% 和 21.5%, ANI 模型的结果稍小。这些结果与相应的实测结果^[7](回流一再附长度为 6.9,最大回流速度为 22%)非常接近。相应的时均速度及各雷诺应力分量与实测值的对比见图 3(□文献[7]实验结果, —ANI 模型结果, ⋯STD 模型结果)。由

图可见,二模型皆给出了合理的时均速度;就整个流场而言,由 STD 模型计算的时均速度较膨胀比为 8:1 情况下的结果有所改进,对于各雷诺应力分量,仅有 ANI 模型给出合理的结果,STD 模型预测的各应力分量与实测值仍有较大差异。

4 结束语

比较表明,本文给出的低雷诺数非线性 $k-\epsilon$ 模型结果较

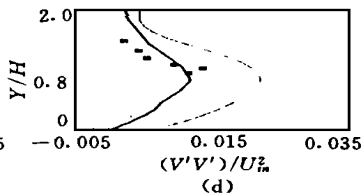
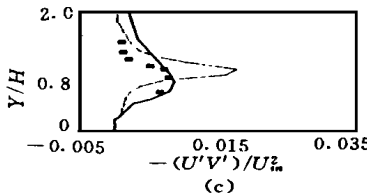
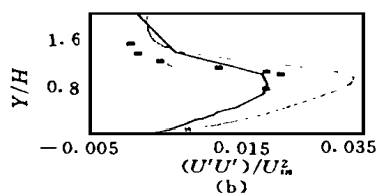
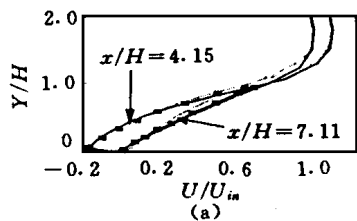


图 3 膨胀比为 2:1 情况下的预测结果与相应实测结果的对比

标准 $k-\epsilon$ 模型结果有明显改进。它对分离再附流动的各特征量包括回流—再附长度、时均速度及各雷诺应力分量均给出了合理的预测。这些结果稍好于有关雷诺应力模型的预测结果。由于本文模型直接应用非滑移壁面边界条件,因而不存在某些湍流模型因采用壁面律函数所带来的误差。

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PREDICTION OF TURBULENT SEPARATED FLOWS WITH A LOW REYNOLDS NUMBER NONLINEAR $K-\epsilon$ MODEL

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ABSTRACT A new low Reynolds number nonlinear $K-\epsilon$ model is presented, in which two correction functions for the near-wall flow and a nonlinear expression for the Reynolds stress are incorporated. Several backstep separated flows with two different expansion ratios are numerically studied by using the above model and a standard $K-\epsilon$ model. In comparison with experimental data, the results obtained show that the predictions of the presented nonlinear model are obviously improved over these of the standard $K-\epsilon$ model in respect of various flow parameters, such as reattachment length, time mean velocities and Reynolds stress components. Also, the presented model is the partial differential Reynolds stress model in prediction of the Reynolds stress components.

Key Words Turbulence characteristics Flow separation Anisotropy field

AN EXPERIMENTAL ANECHOIC RESEARCH OF PERFORATED PLATES FOR NACELLE LINER

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ABSTRACT An experimental anechoic research of perforated plates for nacelle liner has been completed under conditions of different rotor speeds and inlet Mach numbers. The duct, which was lined with single segment or three segments made of the perforate honeycomb plates or had a hard wall, was installed on a transonic axial compressor facility at BUAA. The experimental results discovered some anechoic features of the perforated plate honeycomb structure and enriched data base for designing the engine nacelles. A new nacelle liner, which has larger cavity-depth and open area ratio, has been designed. The largest noise reduction can reach to 10, 5 dB under the condition that the acoustic treatment length is only 30 cm.

Key Words Engine nacelles Perforated plate sound absorption Acoustic experiments

A SYSTEM FOR MEASURING IGNITION DELAY TIME

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ABSTRACT A measuring system for ignition delay time, which consists of an Intel 8253 chip, a microcomputer, an electric magnetic valve and an ultraviolet photodiode was developed. It can realize the real-time and on-line measurement with precision of 0.001 second. The system has been used successfully in measuring of ignition delay time to appraise the performance of a catalytic igniter.

Key Words Catalytic ignition Ignition delay Measuring systems