

高速对称刚性转子碰摩运动的稳定性分析

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【摘要】 分析了高速对称刚性转子圆柱形和圆锥形碰摩运动。当转子两端的轴承同时发生不良润滑,造成转子两端同时与轴承接触,产生圆柱形碰摩运动,给出了发生圆柱形碰摩运动时系统的稳定范围,讨论了摩擦系数、轴承径向间隙、碰摩发生时的初始角度与失稳涡动角速度的关系。当转子一端的轴承发生不良润滑,而另一端的轴承正常工作时发生圆锥形碰摩运动,分析发现其碰摩运动的稳定性取决于其系统本身物理参数,进行了分岔分析,给出了其涡动角速度的一阶近似表达式,这可为转子系统的故障诊断提供一定的理论基础。

主题词: 转子 摩擦 稳定性 分析

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1 引言

在高速旋转机械中,常发生碰摩运动。目前的研究主要集中在瞬态振动分析和稳定性分析上。Choy^[1]分析了转子碰摩时的非线性瞬态振动,Ehrich^[2~4]给出高阶亚谐响应和边频现象,Kim^[5]给出了一Jeffcott转子的伪周期响应和稳定性分析,文献[6~8]分别给出了转子碰摩的振动故障特征分析和混沌现象。张文^[9]给出几种碰摩的稳定性分析。本文给出了对称刚性转子碰摩运动的稳定性分析,分析了发生圆柱形碰摩运动时系统的稳定范围;对于圆锥形碰摩运动,给出了其涡动角速度的表达式。

2 圆柱形碰摩运动

当转子两端的轴承同时发生不良润滑,造成转子两端同时与轴承接触,如图1所示。

转子发生碰摩的条件为:

$$N = mg\cos\theta + mco^2 \geq 0 \quad (1)$$

转子在切线方向的运动方程为:

$$mco^2 + mg\sin\theta + \mu N = 0 \quad (2)$$

在碰摩过程中,假设转子的

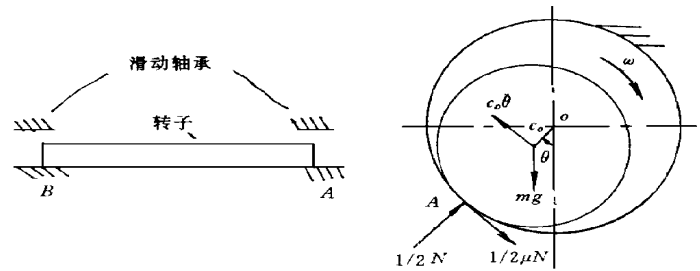


图1 圆柱形碰摩运动

自转角速度由于外界不断供给能量而保持为一个常数。

$$\text{将(1)式代入(2)式得到: } m c_0 \ddot{\theta} + m g \sin \theta + \mu m (g \cos \theta + c_0 \dot{\theta}^2) = 0 \quad (3)$$

此式即为刚性转子圆柱形碰摩运动方程,对(3)式进行简化得:

$$\ddot{\theta} + \Omega^2 (\sin \theta + \mu \cos \theta) + \mu \dot{\theta}^2 = 0 \quad (4)$$

其中 $\Omega^2 = g/c_0$ 。上式中有一个平衡点: $\dot{\theta} = 0, \tan \theta = -\mu, \theta_0 = \tan^{-1}(-\mu)$ 。令 $\theta = \varphi + \theta_0$, (4) 式变为 $\ddot{\varphi} + \mu \dot{\varphi} + \Omega^2 \frac{1 + \mu^2}{1 + 4\mu^2} \sin \varphi = 0$ 。它是一个非线性方程,可令 $y = \dot{\varphi}$ 通过常微分方程的方法进行求解得:

$$\dot{\varphi} = - \left[(2\Omega^2 \frac{1 + \mu^2}{1 + 4\mu^2}) / (1 + 4\mu^2) \right] (2\mu \sin \varphi \cos \varphi + c e^{-2\mu \varphi}) \quad (5)$$

这里 c 是一积分常数,它的值可由初始条件决定。设系统碰摩时的初始条件为 $\varphi(0) = \varphi_0, \dot{\varphi}(0) = \dot{\varphi}_0$, 代入(5)式得到:

$$c = \left[\dot{\varphi}_0 + 2\Omega^2 \frac{1 + \mu^2}{(1 + \mu^2)/(1 + 4\mu^2) \sin(\varphi_0 + \delta)} \right] e^{2\mu \varphi_0} \quad (6)$$

其中: $\delta = -\tan^{-1}(1/2\mu)$ 。将(6)式代入(5)式得到:

$$\dot{\varphi} = - \frac{2\Omega^2 \frac{1 + \mu^2}{1 + 4\mu^2} \sin(\varphi_0 + \delta)}{1 + 4\mu^2} + \left[\dot{\varphi}_0 + 2\Omega^2 \frac{1 + \mu^2}{(1 + \mu^2)/(1 + 4\mu^2) \sin(\varphi_0 + \delta)} \right] e^{2\mu(\varphi - \varphi_0)} \quad (7)$$

上式中解曲线在相平面 $(\varphi, \dot{\varphi})$ 上或者形成封闭曲线或者非封闭曲线。封闭曲线对应着周期解是稳定的,非封闭曲线对应着无界解,分析得到初始条件的失稳区是:

$$\dot{\varphi}_0 / \Omega^2 \geq \frac{(1 + \mu^2)/(1 + 4\mu^2)}{2\sin(\pi + \delta)} e^{-2\mu(\varphi_0 - \pi)} - 2\sin(\varphi_0 + \delta) \quad (8)$$

根据(8)式和(1)式可以看出,失稳阈值 $\dot{\varphi}_0$ 是摩擦系数 μ , 轴承间隙 c_0 和碰摩发生时的初始角度 φ_0 的函数。图2和3给出了数值算例。算例表明,在一定的轴承间隙 c_0 下,刚性转子碰

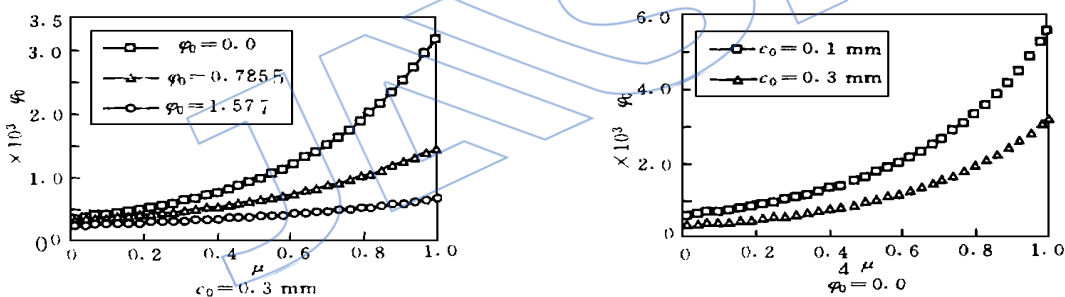


图2 圆柱形碰摩运动的稳定区域

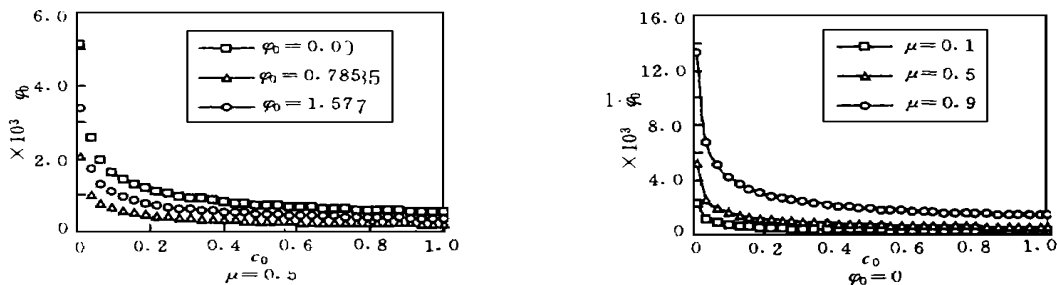


图3 圆柱形碰摩运动的稳定区域

摩的失稳涡动角速度 $\dot{\varphi}$ 随摩擦系数 μ 的增加而增大,随碰摩发生时的初始角度 φ_0 的增加而减小。在一定的摩擦系数 μ 下,刚性转子碰摩的失稳涡动角速度 $\dot{\varphi}$ 随轴承间隙 c_0 的增加而减小,

随碰摩发生时的初始角度 ϑ 的增加而减小。

3 圆锥形碰摩运动

当转子一端的轴承发生不良润滑, 而另一端的轴承正常工作时发生的碰摩运动, 如图 4 所示。选取 $oxyz$ 为静坐标系, $o\xi\eta\zeta$ 为固连于刚体上的动坐标系。

刚体转子绕 O 点的角速度为: $\vec{\Omega} = (\omega - \theta \cos \alpha) \vec{\xi} - \theta \sin \alpha \vec{\eta}$; 动量矩为: $\vec{H}_0 = J_p (\omega - \theta \cos \alpha) \vec{\xi} - J_d \theta \sin \alpha \vec{\eta}$ 。其中 J_d, J_p 分别是刚性转子绕 O 点的赤道转动惯量和极转动惯量。

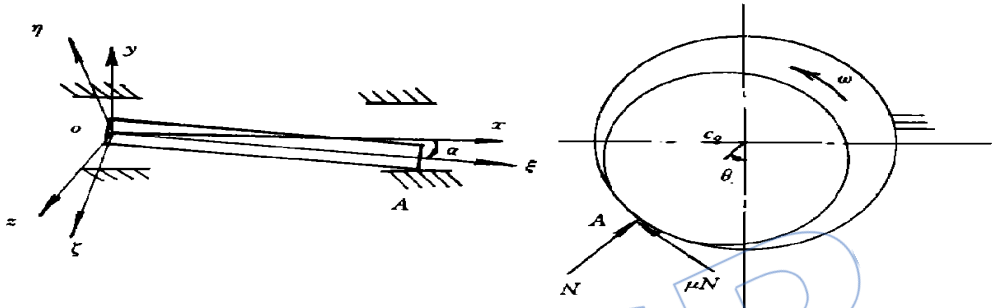


图 4 圆锥形碰摩运动

转子所受外力(重力, 碰摩时产生的摩擦力和正压力)对 O 点的力矩为:

$$\vec{M}_0 = \mu R N \vec{\xi} + [-\mu N l + (1/2) m g l \sin \theta] \vec{\eta} + [N l \cos \alpha - (1/2) m g l \cos \theta \cos \alpha] \vec{\zeta}$$

根据动量矩定理, 并假设在碰摩过程中, 转子的自转角速度为一定值, 保持不变, 且假设 α 是小量, 令 $\sin \alpha \approx c_0/l$, $\cos \alpha \approx 1.0$, 整理得到:

$$N = [(J_d - J_p)/l^2] c_0 \dot{\theta}^2 - [(J_d - J_p)/l^2] c_0 \omega \dot{\theta} + (1/2) m g \cos \theta \quad (9)$$

$$\ddot{\theta} - \mu a \dot{\theta}^2 + \mu a \omega \dot{\theta} + (m g l^2 / 2 J_d c_0) (-\mu \cos \theta + \sin \theta) = 0 \quad (10)$$

(10) 式即为圆锥形碰摩运动的方程, 其 $a = (J_d - J_p)/J_d$ 。必须指出, 它仅当 $\dot{\theta}_0 < R\omega$ 时成立, 否则摩擦力方向相反, μ 取负值。由 $\dot{\theta} = 0$ 得平衡位置是 $\tan \theta = \mu$, $\theta_0 = \tan^{-1} \mu$ 。取相对于平衡位置的扰动角 $y = \theta - \theta_0$ 作广义坐标, 代入得: $y'' - \mu a y'^2 + \mu a \omega y' + (m g l^2 / 2 J_d c_0) \sin y = 0$ (11)

(11) 式是一非线性方程, 令 $T = \omega$, $c = m g l^2 / 2 J_d c_0 \omega$ 引入小参数 $\epsilon = 1/\omega$ 则上式变为:

$$y'' - \mu a y'^2 + \mu a y' + \epsilon c \sin y = 0 \quad (12)$$

$$\text{令 } y = y_0 + \epsilon y_1 + \epsilon^2 y_2 + \dots \quad (13)$$

采用多尺度方法分析(12)式, 将(13)式代入(12)式, 并比较 ϵ 的同次幂的系数项得:

$$\epsilon^0: D_0^2 y_0 - \mu a (D_0 y_0)^2 + \mu a (D_0 y_0) = 0 \quad (14)$$

$$\epsilon^1: D_0^2 y_1 + 2 D_0 D_1 y_0 - 2 \mu a (D_0 y_0) (D_0 y_1) - 2 \mu a (D_0 y_0) (D_0 y_1) + \mu a (D_0 y_1) + \mu a (D_1 y_0) + c \sin y_0 = 0 \quad (15)$$

$$\text{令 } \omega = D_0 y_0, (14) \text{ 式变为: } D_0 \omega - \mu a \omega^2 + \mu a \omega = 0 \quad (16)$$

$$(16) \text{ 式的定常解为: } \bar{\omega} = 1.0, \text{ 或 } \bar{\omega} = 0. \text{ 其解为: } \omega = a / [-a + e^{\mu a (T_0 + A(T_1))}] \quad (17)$$

其中 $A(T_1)$ 是独立于 T_0 的积分常数。设系统的初始条件为 $\theta(0) = \bar{\theta}$ 代入(17)式得:

$$\omega = 1.0 / [1.0 + (1.0/\bar{\theta} - 1.0) e^{\mu a T_0}] \quad (18)$$

如果 $a > 0$, 令 $x = \mu a \omega$, 则(16)式变为: $\dot{x} = x^2 - \mu a x$ (19)

以 μa 为参数, 根据文献[10], 方程(19)产生跨临界分岔(Transcritical bifurcation), 其分岔图

见图 5(a)。如果 $a < 0$, 令 $x = (-\mu a) \omega$, 则(16)式变为: $\dot{x} = -x^2 - \mu a x$ (20)

以 $-\mu a$ 为参数, 方程(20)产生跨临界分岔, 其分岔图见图 5(b)。

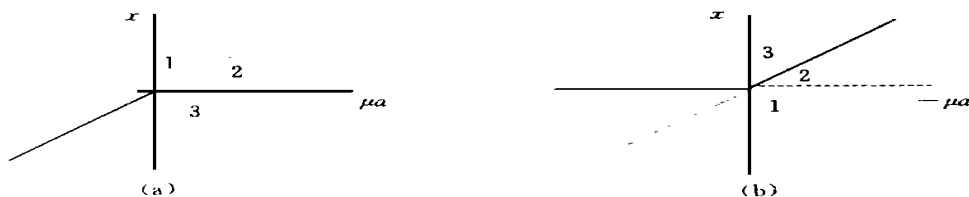


图 5 分岔图

初始条件确定了解是无界的或接近于定常解(对应着固定点)。图 5(a)和(b)说明, 如果碰摩开始时, 系统的参数位于 1 区域, 则碰摩是不稳定的, 其涡动的角速度将发散; 如果系统的参数位于 2, 3 区域, 则碰摩是稳定的, 其稳态涡动的角速度将收敛于 0。(当 $a > 0$ 时)或收敛于 1.0(当 $a < 0$ 时)。在 ω 是稳定定常解的情况下, $D_0 y_0 = \omega$, (15)式可简化为:

$$D_0^2 y_1 - \mu a (2\omega - 1.0) (D_0 y_1) + c \sin \omega T_0 = 0 \quad (21)$$

令 $D_0 y_1 = \omega$, 由(21)式可解得: $\omega = (c / (f^2 + \omega^2)) [\cos(\omega T_0 + \gamma) + e^{-f(T_0 + B(T_1))}]$ (22)

其中 $f = \mu a (1.0 - 2\omega)$, $\tan \gamma = - (f / \omega)$, $B(T_1)$ 是独立于 T_0 的积分常数。显然要是 ω 稳定, 如 $a > 0$, 则 $\omega \leq 1/2$, 如 $a < 0$, 则 $\omega \geq 1/2$ 。

令初值 $\omega(0) = 0$, 代入(22)式得到 $\omega = (c / (f^2 + \omega^2)) [\cos(\omega T_0 + \gamma) - \cos \gamma e^{-f T_0}]$, 由此得到碰摩时其涡动角速度的一阶近似为:

$$\omega = \omega + \epsilon \omega \quad (23)$$

结论 通过对高速对称刚性转子圆柱形碰摩运动和圆锥形碰摩运动的分析, 发现圆柱形碰摩运动的失稳阈值 ω 是摩擦系数 μ 、轴承间隙 c_0 和碰摩发生时的初始角度 φ 的函数。圆锥形碰摩运动的收敛情况, 取决于其系统本身物理系数, 给出了其稳态碰摩时其涡动角速度的一阶近似表达式, 这可为转子系统的安全运行保障条件和故障诊断提供一定的理论基础。

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tion describing relations between the displacements in the centers and torsional angular displacements of the mating gears is derived. The coupled lateral and torsional vibration model of a rotor system with a gear coupling is developed with Lagrange's Equation and the constraint equation in rotating reference frames. The system of two Jeffcott rotors connected by a gear coupling has been analyzed numerically and the results show that the transient response frequency of aligned rotors approximately equals to the uncoupled model in lateral vibration, but the even- integer multiples of lateral vibration and the odd- integer multiples of torsional vibration are found out by the coupled model for the misaligned rotors.

Key Words Rotors Gears Couplings Vibrations

STABILITY OF RUBBING MOTIONS OF SYMMETRIC RIGID ROTOR IN HIGH SPEED ROTATING MACHINERY

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ABSTRACT

The cylindrical and conical rubbing motions of a symmetric rigid rotor in high speed rotating machinery are investigated. When both journal bearings of the rotor are simultaneously under starvation lubrication condition, both ends of the rotor contact locally with the bearings so that the cylindrical rubbing motions occur. The stability margin of the cylindrical rubbing motions is determined, and the relationships of instability with the radial clearance, the bearing/rotor friction coefficients, and the angle of incidence during successive rubs are obtained. When one of the journal bearings is under starvation lubrication condition, the conical rubbing motion of the rotor occurs. The analysis shows that in this case the stability of rotor depends on its physical parameters, the transcritical bifurcation is analyzed. Then the first-order approximation formula of whirling angular speed is given, which provides theoretical analysis for diagnosis of rotor systems.

Key Words Rotors Friction Stability Analysis

SIMULATION OF DOPPLER ON- LINE MONITORING TECHNIQUE FOR BLADE CRACK FAULT

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ABSTRACT

An on- line monitoring technique for blade crack fault is set up according to the mechanism of an acoustic doppler on- line monitoring technique, which is based on cracked blade's resonant phenomenon in turbine machinery. But the resonant phenomenon can not be captured easily, because the fatigue crack expands very quickly due to the high frequency vibration of turbine blade. So that a new monitoring method based on the unsymmetry in vibration response of the cracked blade is developed, which utilizes the achievements in analyzing the vibration of the cracked blade. The program for calculating vibration response wave form of the rotating blade from acoustic doppler signals is worked out. Simula-