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两相湍流的非线性 $k-\epsilon-k_p$ 模型

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摘要: 由两相湍流的代数应力模型, 提出一种两相湍流的非线性 $k-\epsilon-k_p$ 模型, 可以合理地模拟各向异性较强的旋流, 浮力流等两相流动, 具备二阶矩模型的优点, 又比二阶矩模型简单。文中推导出气相、颗粒相的雷诺应力和两相脉动速度关联的非线性应力应变关系式。这些代数式和两相各自的湍动能 k , k_p 以及两相脉动关联湍动能 k_{pg} 的方程联立, 就构成非线性 $k-\epsilon-k_p$ 模型。本文将该模型用于模拟突扩湍流两相流动, 给出两相时均速度场及雷诺应力各分量, 并且将模拟结果和 PDPA 实验数据以及二阶矩模型的模拟结果对照。结果表明, 该模型预报各向异性两相湍流的能力和二阶矩模型的能力接近, 但是计算量比二阶矩模型的小得多。

关键词: 两相流; 湍流模型; 非线性模型

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1 引言

周力行^[1]提出了两相湍流的 $k-\epsilon-k_p$ 模型和统一二阶矩模型(USM)。在应用中已发现线性的 $k-\epsilon-k_p$ 模型能较好地模拟无旋及弱旋流动, 但是对于强旋流动等强各向异性湍流两相流动, USM 模型能给出较合理结果。然而 USM 模型比较复杂, 应用于工程实际问题, 如三维两相流动和两相燃烧的数值模拟中, 计算量太大。综合考虑模型的合理性及经济性, 可以选择折中的两相湍流的非线性 $k-\epsilon-k_p$ 模型和代数应力模型(ASM)。但是数值模拟的经验表明, ASM 模型的动量方程右端缺少扩散项而仅有源项, 尽管在数值方法上采取种种措施, 在计算中, 特别是三维问题中, 经常发散, 难以得到收敛结果。如果使用非线性 $k-\epsilon-k_p$ 模型, 由于其动量方程和 k , ϵ , k_p 方程在形式上与标准 $k-\epsilon-k_p$ 模型的相应方程基本相同, 因而易于得到收敛结果, 而非线性的应力应变关系同样可以反映各向异性湍流的特征。因而有必要发展非线性的 $k-\epsilon-k_p$ 模型。

单相湍流流动的非线性 $k-\epsilon$ 模型的研究已经

取得了相当进展^[2~8], 被认为是当前工程中有前途的湍流模型之一。非线性的应力应变关系可由以下几种方法得到: (1) 理性力学的方法(量纲分析和不变性原理)^[2,3]; (2) 重整化群(RNG)方法^[4]; (3) 双尺度直接相互作用近似^[5]; (4) 由代数应力模型推导得出^[6]。在单相湍流的非线性应力应变关系式中, 等价的湍流粘性系数不再是标量, 而是各向异性的张量, 所以该非线性模型能模拟旋流、分离流动、再附点流动中的湍流各向异性效应。非线性 $k-\epsilon$ 模型已经在简单流动(均匀剪切流和管道流动)中得到检验, 并已应用于复杂流动(后台阶流动、旋流流动等)^[2,8], 与实验数据的对比显示出此模型较标准的各向同性的 $k-\epsilon$ 模型有较大改进, 而二者的计算量相当。这说明非线性 $k-\epsilon$ 模型用于工程实际问题有较好的前景。

本文由两相湍流代数应力模型表达式, 提出两相湍流的非线性 $k-\epsilon-k_p$ 模型, 将两相湍流的非线性 $k-\epsilon-k_p$ 模型用于模拟突扩湍流两相流动, 给出两相时均速度场和雷诺应力各分量, 并将所得结果和 USM 模型, 线性 $k-\epsilon-k_p$ 模型的模拟结果以及 PDPA 实验结果对照, 对模型进行评价。

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2 两相湍流的非线性应力应变关系式

建立两相湍流的非线性应力应变关系式的出

$$\overline{v_i v_j} = (1 - \lambda) \frac{2}{3} k \delta_{ij} + \lambda \frac{k}{\epsilon} \left[\overline{v_i v_k} \frac{\partial v_j}{\partial x_k} + \overline{v_j v_k} \frac{\partial v_i}{\partial x_k} \right] + \frac{k \rho_p}{c_1 \rho \epsilon \tau_p} \left(\overline{v_{pi} v_j} + \overline{v_i v_{pj}} - 2 \overline{v_i v_j} \right) \quad (1)$$

$$\overline{v_{pi} v_{pj}} = - \frac{\tau_p}{2} \left[\overline{v_{pi} v_{pk}} \frac{\partial v_{pj}}{\partial x_k} + \overline{v_{pj} v_{pk}} \frac{\partial v_{pi}}{\partial x_k} \right] + \frac{1}{2} \left(\overline{v_i v_{pj}} + \overline{v_{pi} v_j} \right) \quad (2)$$

$$\overline{v_{pi} v_j} = - \frac{\rho \tau_p}{\rho + \rho_p} \left[\overline{v_{pi} v_k} \frac{\partial v_j}{\partial x_k} + \overline{v_j v_{pk}} \frac{\partial v_{pi}}{\partial x_k} \right] + \frac{\rho}{\rho + \rho_p} \overline{v_i v_j} + \frac{\rho_p}{\rho + \rho_p} \overline{v_{pi} v_{pj}} - \frac{\rho \tau_p}{\rho + \rho_p} \frac{\epsilon}{k} \overline{v_{pi} v_i} \delta_{ij} \quad (3)$$

把上述表达式整理成应力和应变关系的显式, 得到如下的关系式:

$$\overline{v_i v_j} = A(1) \delta_{ij} + A(2) \left[\overline{v_i v_k} \frac{\partial v_j}{\partial x_k} + \overline{v_j v_k} \frac{\partial v_i}{\partial x_k} \right] + A(3) \left[\overline{v_{pi} v_{pk}} \frac{\partial v_{pj}}{\partial x_k} + \overline{v_{pj} v_{pk}} \frac{\partial v_{pi}}{\partial x_k} \right] + A(4) \left[\overline{v_{pi} v_k} \frac{\partial v_j}{\partial x_k} + \overline{v_j v_{pk}} \frac{\partial v_{pi}}{\partial x_k} + \overline{v_i v_{pk}} \frac{\partial v_{pj}}{\partial x_k} + \overline{v_{pj} v_k} \frac{\partial v_i}{\partial x_k} \right] \quad (4)$$

$$\overline{v_{pi} v_{pj}} = B(1) \delta_{ij} + B(2) \left[\overline{v_i v_k} \frac{\partial v_j}{\partial x_k} + \overline{v_j v_k} \frac{\partial v_i}{\partial x_k} \right] + B(3) \left[\overline{v_{pi} v_{pk}} \frac{\partial v_{pj}}{\partial x_k} + \overline{v_{pj} v_{pk}} \frac{\partial v_{pi}}{\partial x_k} \right] + B(4) \left[\overline{v_{pi} v_k} \frac{\partial v_j}{\partial x_k} + \overline{v_j v_{pk}} \frac{\partial v_{pi}}{\partial x_k} + \overline{v_i v_{pk}} \frac{\partial v_{pj}}{\partial x_k} + \overline{v_{pj} v_k} \frac{\partial v_i}{\partial x_k} \right] \quad (5)$$

$$\overline{v_{pi} v_j} = C(1) \delta_{ij} + C(2) \left[\overline{v_i v_k} \frac{\partial v_j}{\partial x_k} + \overline{v_j v_k} \frac{\partial v_i}{\partial x_k} \right] + C(3) \left[\overline{v_{pi} v_{pk}} \frac{\partial v_{pj}}{\partial x_k} + \overline{v_{pj} v_{pk}} \frac{\partial v_{pi}}{\partial x_k} \right] + C(4) \left[\overline{v_{pi} v_{pk}} \frac{\partial v_{pj}}{\partial x_k} + \overline{v_{pj} v_k} \frac{\partial v_i}{\partial x_k} \right] + C(5) \left[\overline{v_{pi} v_k} \frac{\partial v_j}{\partial x_k} + \overline{v_j v_{pk}} \frac{\partial v_{pi}}{\partial x_k} \right] \quad (6)$$

其中: $A(1) \sim A(4), B(1) \sim B(4), C(1) \sim C(5)$ 等均为 $k, \epsilon, k_p, k_{pg}, \rho_p, \tau_p$ 的函数, 详见表1。

表1 函数 A, B, C 的表达式

	A	B	C
1	$(1 - \lambda) \frac{2}{3} k - \frac{2 \rho_p}{c_1 \rho} \frac{2}{3} k_{pg}$	$(1 - \lambda) \frac{2}{3} k - \left(\tau_p \frac{\epsilon}{k} + \frac{2 \rho_p}{c_1 \rho} \right) \frac{2}{3} k_{pg}$	
2	$-\lambda(k/\epsilon)$	$-\lambda(k/\epsilon)$	$-\lambda(k/\epsilon)$
3	$-\frac{\rho_p \tau_p}{\rho}$	$-\frac{\tau_p}{2 \rho} (\rho + \rho_p + 2M \rho_p)$	$-\frac{\rho_p \tau_p}{\rho} \left(\frac{1}{2} + M \right)$
4	$-M \tau_p$	$-\tau_p \left(M + \frac{1}{2} \right)$	$-\tau_p \left(M + \frac{\rho_p}{2(\rho + \rho_p)} \right)$
5			$-\tau_p \left(M + \frac{\rho_p}{2(\rho + \rho_p)} + \frac{\rho_p}{\rho + \rho_p} \right)$

表中: $M = \frac{k \rho_p}{c_1 \rho \epsilon \tau_p}$, $\tau_p = \frac{d_p^2 \bar{\rho}_p}{18 \mu} \left(1 + \frac{Re_p^{2/3}}{6} \right)^{-1}$, $Re_p = \frac{|\vec{V}_p - \vec{V}| d_p}{\nu}$, $\lambda = \frac{1 - c_2}{c_1}$

其中: k 为气相湍动能, ϵ 为 k 的耗散率, k_p 为颗粒相湍动能, k_{pg} 为两相脉动关联的湍动能, $k_{pg} = \overline{v_i v_{pi} v_j}$ 上述变量均由守恒方程得出。

将 $\overline{v_m v_k}, \overline{v_{pm} v_{pk}}, \overline{v_{pm} v_k}, \overline{v_m v_{pk}}, (m = i, j)$ 用相同形式表达出来, 然后将其代入 $\overline{v_i v_j}, \overline{v_{pi} v_{pj}}, \overline{v_{pi} v_j}$ 的表达式, 得出如下形式的非线性应力应变关系式:

$$\overline{v_i v_j} = C \delta_{ij} + O \left[\frac{\partial v_m}{\partial x_n} \right] + O \left[\left[\frac{\partial v_m}{\partial x_n} \right]^2 \right] + O \left[\left[\frac{\partial v_m}{\partial x_n} \right]^3 \right] + \cdots + O \left[\left[\frac{\partial v_m}{\partial x_n} \right]^\infty \right] \quad C \text{ 为常数} \quad (7)$$

实际应用中, 必须截取一定项数。单相流动中, Shih T H 和 Hirsch 建议, 对于剪切流动和旋流, 系数适当的二次项已能给出较好预报结果。这里仅给出保留到应变率的二次项的非线性应力应

发点是两相湍流代数应力模型的气相、颗粒相雷诺应力和两相脉动速度关联的表达式(式中, $\overline{v_i v_j}$ 和 $\overline{v_{pi} v_{pj}}$ 分别为气相和颗粒相雷诺应力, $\overline{v_{pi} v_j}$ 是两相脉动速度关联):

变关系式:

$$\begin{aligned} \overline{v_i v_j} = & G_1 \delta_{ij} + G_2 \left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right] + G_3 \left[\frac{\partial v_{pi}}{\partial x_j} + \frac{\partial v_{pj}}{\partial x_i} \right] + G_8 \left[\left(\frac{\partial v_i}{\partial x_k} - \frac{\partial v_{pi}}{\partial x_k} \right) \left(\frac{\partial v_j}{\partial x_k} - \frac{\partial v_{pj}}{\partial x_k} \right) \right] + \\ & G_4 \left[\frac{\partial v_i}{\partial x_k} \left(\frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} \right) + \frac{\partial v_j}{\partial x_k} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right) \right] + G_5 \left[\frac{\partial v_{pi}}{\partial x_k} \left(\frac{\partial v_{pj}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_j} \right) + \frac{\partial v_{pj}}{\partial x_k} \left(\frac{\partial v_{pi}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_i} \right) \right] + \\ & G_6 \left[\frac{\partial v_i}{\partial x_k} \left(\frac{\partial v_{pj}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_j} \right) + \frac{\partial v_j}{\partial x_k} \left(\frac{\partial v_{pi}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_i} \right) \right] + G_7 \left[\frac{\partial v_{pi}}{\partial x_k} \left(\frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} \right) + \frac{\partial v_{pj}}{\partial x_k} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right) \right] \end{aligned} \quad (8)$$

$$\begin{aligned} \overline{v_{pi} v_{pj}} = & P_1 \delta_{ij} + P_2 \left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right] + P_3 \left[\frac{\partial v_{pi}}{\partial x_j} + \frac{\partial v_{pj}}{\partial x_i} \right] + P_8 \left[\left(\frac{\partial v_i}{\partial x_k} - \frac{\partial v_{pi}}{\partial x_k} \right) \left(\frac{\partial v_j}{\partial x_k} - \frac{\partial v_{pj}}{\partial x_k} \right) \right] + \\ & P_4 \left[\frac{\partial v_i}{\partial x_k} \left(\frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} \right) + \frac{\partial v_j}{\partial x_k} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right) \right] + P_5 \left[\frac{\partial v_{pi}}{\partial x_k} \left(\frac{\partial v_{pj}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_j} \right) + \frac{\partial v_{pj}}{\partial x_k} \left(\frac{\partial v_{pi}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_i} \right) \right] + \\ & P_6 \left[\frac{\partial v_i}{\partial x_k} \left(\frac{\partial v_{pj}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_j} \right) + \frac{\partial v_j}{\partial x_k} \left(\frac{\partial v_{pi}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_i} \right) \right] + P_7 \left[\frac{\partial v_{pi}}{\partial x_k} \left(\frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} \right) + \frac{\partial v_{pj}}{\partial x_k} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right) \right] \end{aligned} \quad (9)$$

$$\begin{aligned} \overline{v_{pi} v_j} = & T_1 \delta_{ij} + T_2 \left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right] + T_3 \left[\frac{\partial v_{pi}}{\partial x_j} + \frac{\partial v_{pj}}{\partial x_i} \right] + T_4 \left[\frac{\partial v_i}{\partial x_k} + \frac{\partial v_{pi}}{\partial x_k} \right] + T_5 \left[\frac{\partial v_j}{\partial x_k} + \frac{\partial v_{pj}}{\partial x_k} \right] + \\ & T_6 \left[\frac{\partial v_i}{\partial x_k} \left(\frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} \right) + \frac{\partial v_j}{\partial x_k} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right) \right] + T_7 \left[\frac{\partial v_{pi}}{\partial x_k} \left(\frac{\partial v_{pj}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_j} \right) + \frac{\partial v_{pj}}{\partial x_k} \left(\frac{\partial v_{pi}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_i} \right) \right] + \\ & T_8 \left[\frac{\partial v_{pi}}{\partial x_k} \left(\frac{\partial v_{pj}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_j} \right) + \frac{\partial v_{pj}}{\partial x_k} \left(\frac{\partial v_{pi}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_i} \right) \right] + T_9 \left[\frac{\partial v_{pi}}{\partial x_k} \left(\frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} \right) + \frac{\partial v_{pj}}{\partial x_k} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right) \right] + \\ & T_{10} \left[\frac{\partial v_{pi}}{\partial x_k} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right) + \frac{\partial v_{pj}}{\partial x_k} \left(\frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} \right) \right] + T_{11} \left[\frac{\partial v_i}{\partial x_k} \left(\frac{\partial v_{pj}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_j} \right) + \frac{\partial v_j}{\partial x_k} \left(\frac{\partial v_{pi}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_i} \right) \right] + \\ & T_{12} \left[\frac{\partial v_i}{\partial x_k} \left(\frac{\partial v_{pj}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_j} \right) + \frac{\partial v_j}{\partial x_k} \left(\frac{\partial v_{pi}}{\partial x_k} + \frac{\partial v_{pk}}{\partial x_i} \right) \right] + T_{13} \left[\frac{\partial v_i}{\partial x_k} \left(\frac{\partial v_{pj}}{\partial x_k} - \frac{\partial v_j}{\partial x_k} \right) + \frac{\partial v_{pj}}{\partial x_k} \left(\frac{\partial v_i}{\partial x_k} - \frac{\partial v_{pi}}{\partial x_k} \right) \right] + T_{14} \left[\frac{\partial v_i}{\partial x_k} \left(\frac{\partial v_{pj}}{\partial x_k} - \frac{\partial v_j}{\partial x_k} \right) + \frac{\partial v_{pj}}{\partial x_k} \left(\frac{\partial v_i}{\partial x_k} - \frac{\partial v_{pi}}{\partial x_k} \right) \right] \end{aligned} \quad (10)$$

其中: $G_1 \sim G_8, P_1 \sim P_8, T_1 \sim T_{14}$, 均为 $k, \epsilon, k_p, k_{pg}, \rho, \rho_p, \tau_p$ 的函数

$$G_1 = A(1); \quad G_2 = A(2)A(1) + A(4)C(1); \quad G_3 = A(3)B(1) + A(4)C(1)$$

$$G_4 = A(1)[A(2)A(2) + A(4)C(2)] + C(1)[A(2)A(4) + A(4)C(5)]$$

$$G_5 = B(1)[A(3)B(3) + A(4)C(3)] + C(1)[A(3)B(4) + A(4)C(5)]$$

$$G_6 = B(1)[A(2)A(3) + A(4)C(3)] + C(1)[A(2)A(4) + A(4)C(4)]$$

$$G_7 = A(1)[A(3)B(2) + A(4)C(2)] + C(1)[A(3)B(4) + A(4)C(4)]$$

$$G_8 = 2C(1)A(4)[C(4) - C(5)]$$

上述非线性应力应变关系式能反映两相湍流的各向异性。此关系式不同于标准 $k-\epsilon k_p$ 模型中的线

性应力应变关系: $\overline{v_i v_j} = \frac{2}{3} k \delta_{ij} - \nu_i \left[\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right], \quad \overline{v_{pi} v_{pj}} = -\nu_p \left[\frac{\partial v_{pj}}{\partial x_i} + \frac{\partial v_{pi}}{\partial x_j} \right]$

3 k, ϵ, k_p 和 k_{pg} 方程

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_k}(\rho v_k k) = \frac{\partial}{\partial x_k} \left(\frac{\mu_e}{\sigma_k} \frac{\partial k}{\partial x_k} \right) - \rho \overline{v_i v_k} \frac{\partial v_i}{\partial x_k} - \rho \epsilon + \frac{\rho_p}{\tau_p} (2k_{pg} - 2k) \quad (11)$$

$$\frac{\partial}{\partial t}(\rho \epsilon) + \frac{\partial}{\partial x_k}(\rho v_k \epsilon) = \frac{\partial}{\partial x_k} \left(\frac{\mu_e}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_k} \right) + \frac{\epsilon}{k} \left\{ c_{e1} \left[-\rho \overline{v_i v_k} \frac{\partial v_i}{\partial x_k} + \frac{\rho_p}{\tau_p} (2k_{pg} - 2k) \right] - c_{e2} \rho \epsilon \right\} \quad (12)$$

$$\frac{\partial}{\partial t}(N_p k_p) + \frac{\partial}{\partial x_k}(N_p v_{pk} k_p) = \frac{\partial}{\partial x_k} \left(N_p \nu_p \frac{\partial k_p}{\partial x_k} \right) - N_p \overline{v_{pi} v_{pk}} \frac{\partial v_{pi}}{\partial x_k} - N_p \epsilon_p \quad (13)$$

$$\begin{aligned} \frac{\partial}{\partial t}(k_{pg}) + (V_k + V_{pk}) \frac{\partial}{\partial x_k}(k_{pg}) = & \frac{\partial}{\partial x_k} \left[(u + u_p) \frac{\partial k_{pg}}{\partial x_k} \right] - \frac{1}{2} \left[\overline{v_i v_{pk}} \frac{\partial v_{pi}}{\partial x_k} + \overline{v_{pi} v_k} \frac{\partial v_i}{\partial x_k} \right] - \\ & \frac{\epsilon}{k} k_{pg} + \frac{1}{\rho \tau_p} [\rho_p k_p + \rho k - (\rho + \rho_p) k_{pg}] \end{aligned} \quad (14)$$

其中: $\epsilon_p = -\frac{1}{N_p \tau_p} [N_p (2k_{pg} - 2k_p) + (V_i - V_{pi}) n_p v_{pi}]$, $\mu_e = \mu + \mu_i$, $\mu_i = c_\mu \rho \frac{k^2}{\epsilon}$, $\nu_p = c_{\mu p} \frac{k_p^2}{\epsilon_p}$

通用常数: $c_\mu = 0.09$, $c_{e1} = 1.45$, $c_{e2} = 1.92$, $\sigma_k = 1.0$, $\sigma_\epsilon = 1.22$

$$c_1 = 3.0, \quad c_2 = 0.3, \quad \sigma_p = 0.7, \quad c_{\mu p} = 0.0064$$

4 突扩两相湍流流动的数值模拟

本文应用非线性的 $k-\epsilon-k$ 两相湍流模型(简称为 NKP)对文献[9]中轴对称两相突扩湍流流动进行了数值模拟。所模拟的通道突扩比为 $\delta/h=0.4$ 。通道几何尺寸和进口流动参数见文献[9]。所用网格数为 20×43 。将上述微分方程组在控制单元内积分,采用混合差分格式,得到差分方程组。对差分方程组用压力—速度修正的 SMPL EC 算法求解。编制了 NKP-2 程序。收敛标准为气相和

颗粒质量余源和小于万分之一。该程序在 Pentium-2 PC 微机上运行一个工况需要 6 m in。而用文献[9]的 USM 程序模拟同一个对象,达到同一收敛标准,运行一个工况需要 10 m in。本模型的模拟结果与实验数据^[9]、线性 $k-\epsilon-k_p$ 模型(简称 KP)及二阶矩模型(USM)的模拟结果^[9]进行了对比。图1为气相轴向时均速度分布,采用这3种模型模拟所得气相平均场均与实验值吻合较好。模拟所得回流区长度约为台阶高度的8倍,都和实验值接近。图2为 $50 \mu\text{m}$ 颗粒的轴向时均速度分

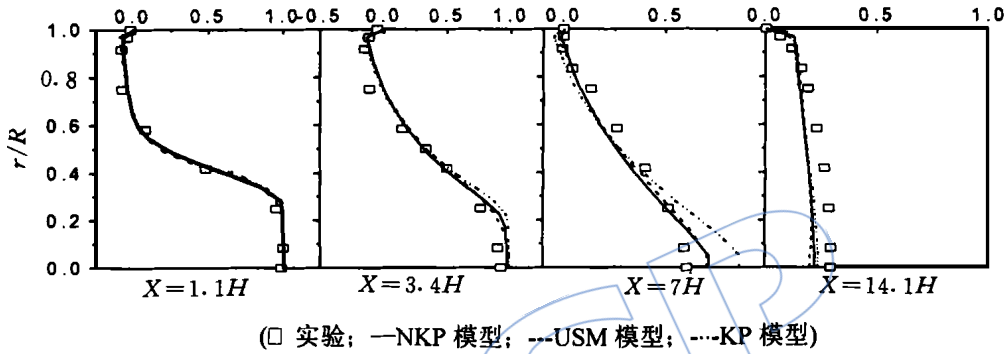


图1 气体轴向时均速度分布(U/U_{in})

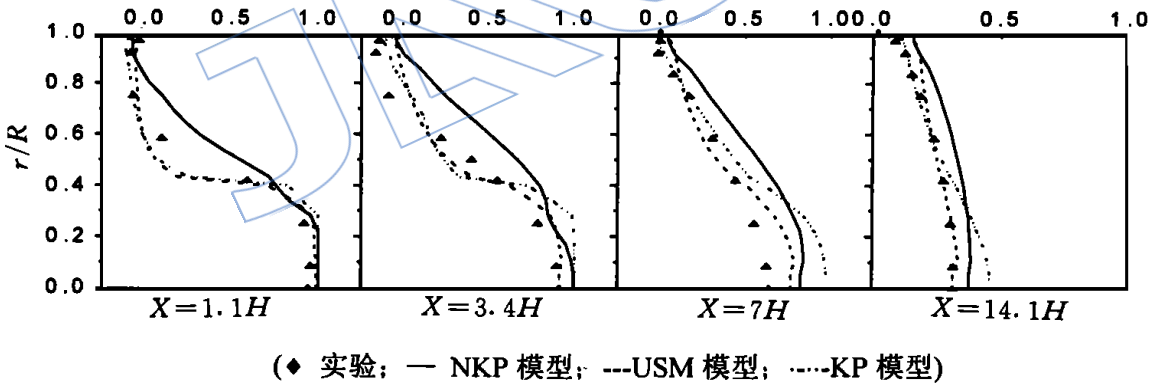


图2 $50 \mu\text{m}$ 颗粒的轴向平均速度分布(U_p/U_{in})

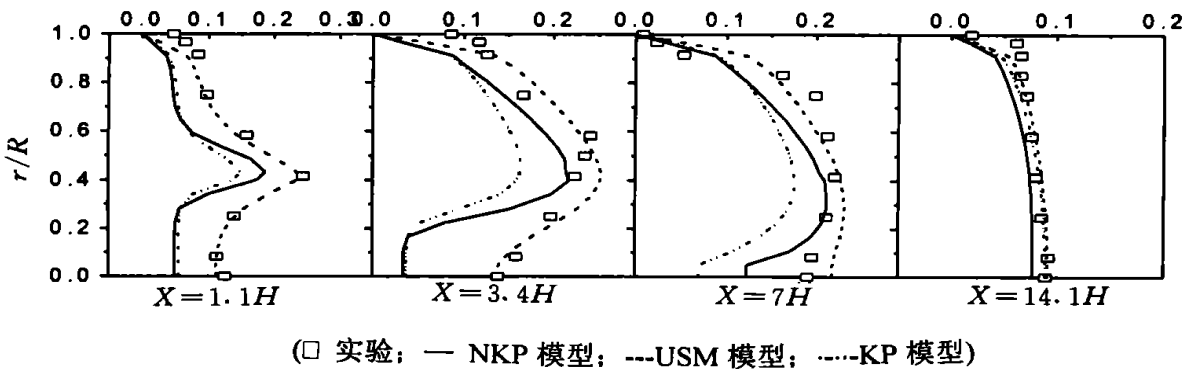


图3 气体轴向脉动速度分布(m/s)

布。NKP 模型模拟结果虽略大于实验值,但总体上与其他两个模型的模拟结果以及实验结果接近。图3给出了气相轴向脉动速度分布,NKP 模型模拟结果和USM 模型模拟结果以及实验值都很接近,而KP 模型模拟结果显著小于实验值,因为KP 模型中的轴向脉动速度是按湍流各向同性,将湍动能乘以2/3再开方求出的。图4给出了气相径向脉动速度分布,NKP 和USM 模型模拟结果都与实验值差距不大,而KP 模型模拟结果显然大于实验值,其原因同前。NKP 模型对50 μm 的

颗粒轴向脉动速度(图5)和径向脉动速度(图6)给出了尚好的模拟结果,与USM 模型比较,NKP 模型能给出剪切层中脉动速度的高峰分布,比USM 模型稍好,但本模型同样存在主流区内模拟所得的轴向脉动速度比实验值偏小的情况。这是因为NKP 模型中,脉动量的表达式是由雷诺应力输运方程出发,采用局部平衡的简化(即产生加耗散等于零)推导得出,未计入对流和扩散的影响,在主流区,因速度梯度小,产生项较小,因而脉动值的模拟量偏小。对颗粒径向脉动速度的

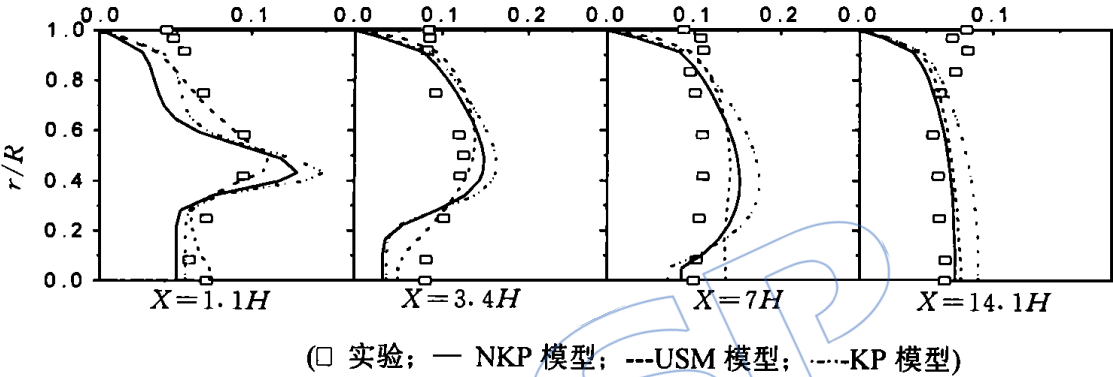


图4 气体径向脉动速度分布

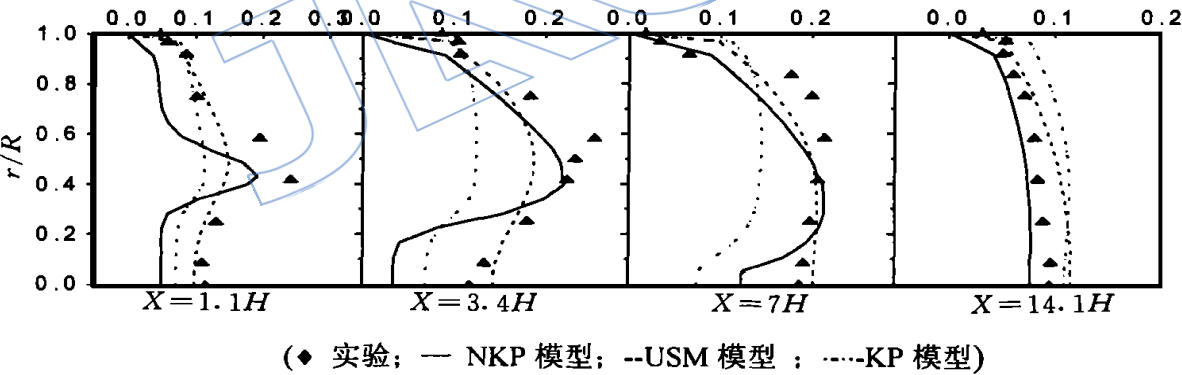


图5 50 μm 颗粒的轴向脉动速度分布

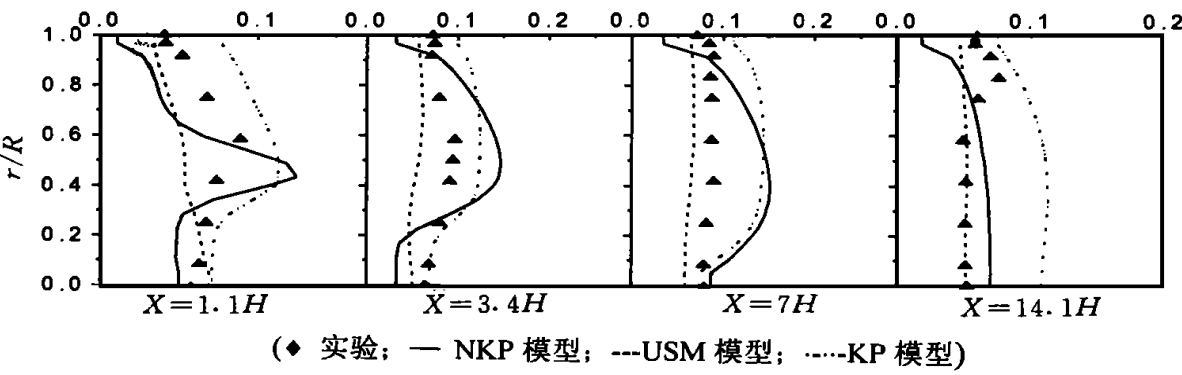


图6 50 μm 颗粒的径向脉动速度分布

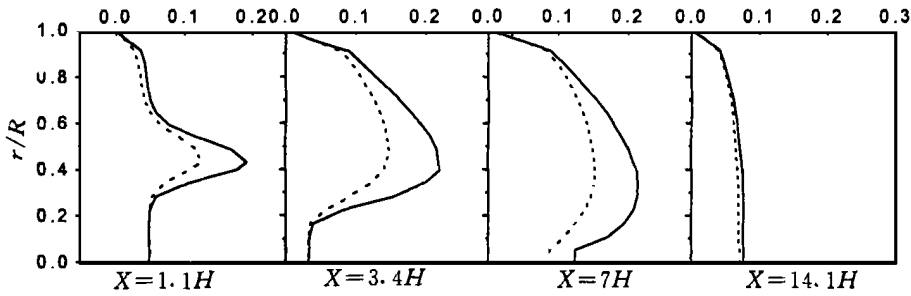


图7 50 μm 颗粒轴向脉动与径向脉动的比较(NKP 模型) — 轴向脉动; -- 径向脉动

模拟,NKP 模型不如USM 模型,但是二者差别比和KP 模型的差别小。KP 模型对颗粒脉动速度的模拟结果有和气相值类似的缺陷。图7为NKP 模型所得的50 μm 颗粒轴向脉动速度与径向脉动速度的比较,两者的差别明显,在剪切层中轴向脉动速度约为径向脉动速度的1.5~2倍。

5 结 论

- (1) 非线性 $k-\epsilon-k_p$ 两相湍流模型和二阶矩模型具有差不多的模拟各向异性两相湍流的能力。
- (2) 该模型能避免代数应力模型应用中经常遇到的不收敛问题。
- (3) 该模型容易耦合到标准 $k-\epsilon-k_p$ 模型的计算程序中,计算时间增加不多,比二阶矩模型所需计算时间少得多。

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A Non-Linear $k-\epsilon-k_p$ Two-Phase Turbulence Model

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Abstract: A nonlinear $k-\epsilon-k_p$ two-phase turbulence model is proposed for simulating anisotropic turbulent two-phase flows, such as swirling and buoyant flows. Non-linear relationships of two-phase stresses and two-phase fluctuation velocity correlations with the strain rates are derived based on the two-phase algebraic Reynolds stress expressions. The nonlinear $k-\epsilon-k_p$ model is applied to simulating two-phase sudden-expansion flows and the results are compared with those obtained from the second-order moment model, linear $k-\epsilon-k_p$ model and experimental data. It is shown that the nonlinear $k-\epsilon-k_p$ model is almost as good as the second-order moment model but with much less computation time, and is obviously better than the linear $k-\epsilon-k_p$ model in simulating anisotropic two-phase turbulence.

Key words: Two-phase flows; turbulence model; non-linear model