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相关系数平稳序列的估计与识别

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摘要: 本文提出了通过观测序列确定不可直接测得的相关系数平稳序列模型参数的方法, 详细讨论了相关系数 $AR(p)$, $MA(q)$ 和 $ARMA(p, q)$ 序列状态方程的参数估计, 为非平稳序列的滤波、预测和平滑提供了依据, 文中还给出了观测系统参数的识别方法, 与传统方法相比, 本文方法考虑了测量噪声的均值和方差随时间变化情况, 具有更高的识别精度。

关键词: 航空、航天推进系统; 相关系数平稳序列; 滤波; 系统辨识; 参数估计; 极大似然估计

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Estimation and Identification for Correlation Coefficient Stationary Series

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Abstract: The estimation method for the parameters of the correlation coefficient stationary series which can not be measured straightly is presented. After the state equations of the correlation coefficient $AR(p)$, $MA(q)$ and $ARMA(p, q)$ series are established, the filtering, prediction and smoothing for non-stationary series can be carried out. In addition, the identification method of the observation system is also given. For the relation between time and the mean, the variance of the noise can be considered, the precision of the present method is higher than that of the traditional methods.

Key words: aerospace propulsion system; correlation coefficient stationary series; filtering; system identification; parameter estimation; maximum likelihood estimation

1 引言

相关系数平稳序列是从非平稳序列中分离出来的一类工程上常见且便于研究的时间序列, 其均值和方差都可随时间变化, 传统的平稳序列只

是它的一个特例。文献[1~3]给出了相关系数 $AR(p)$, $MA(q)$ 和 $ARMA(p, q)$ 序列可直接测得情况下模型参数的估计方法。但是, 在工程实际中许多时间序列是不可直接测得的, 它们往往混杂在噪声中, 例如, 通讯信号由于天气干扰、电磁干

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扰和通讯系统内部干扰,接收时会存在噪音;GPS信号由于SA误差、电离层延迟和对流层延迟误差等影响也会导致定位不准确等。为此,本文首先提出了相关系数AR(p)、MA(q)和ARMA(p,q)序列不可直接测得时的参数估计方法,其次分测量噪声独立和相关两种情况给出了观测系统的参数识别方法。本文方法充分考虑了测量噪声的均值和方差随时间变化的情况,与传统方法相比具有更高的估计和识别精度。

2 相关系数平稳序列参数估计

2.1 基本方程

设 $x_i = x(t_i)$, $t_i = t_0 + i\Delta t$, $i = 1, 2, \dots, n$, 是来自相关系数平稳过程 $X(t)$ 的样本函数的一组观测值,其中 t_0 和 Δt 分别为采样初始时刻和采样间隔。文献[1]建立了相关系数ARMA(p,q)模型

$$\frac{x_k - \mu(a, t_k)}{I(b, t_k)} = \sum_{i=1}^p \frac{\eta[x_{k-i} - \mu(a, t_{k-i})]}{I(b, t_{k-i})} + \epsilon_k - \sum_{i=1}^q \theta_i \epsilon_{k-i} \quad (1)$$

式中 $E[x(t_k)] = \mu(a, t_k)$, $\text{Var}[x(t_k)] = \sigma^2 I^2(b, t_k)$, $E[\epsilon_k] = \sigma^2 \delta_k$, 其中当 $k = i$ 时 $\delta_i = 1$, 当 $k \neq i$ 时, $\delta_i = 0$ 。当 $q = 0$ 时, 式(1)成为相关系数AR(p)模型; 当 $p = 0$ 时, 式(1)则变成相关系数MA(q)模型。文献[1~3]给出了在直接测得相关系数平稳序列 $x_1, x_2, \dots, x_n$ 情况下, 模型参数 $a = (a_0, a_1, \dots, a_r)$; $b = (b_0, b_1, \dots, b_s)$; $\eta = (\eta_1, \eta_2, \dots, \eta_p)$; $\theta = (\theta_1, \theta_2, \dots, \theta_q)$ 和 σ^2 的估计方法。下面将进一步讨论在只能得到其观测序列 $z_1, z_2, \dots, z_n$, 而不能直接测得相关系数平稳序列 $x_1, x_2, \dots, x_n$ 的情况下, 模型参数 a, b, η, θ 和 σ^2 的确定问题。

设观测系统为线性时不变系统, 则有

$$z_k = c_k + \sum_{i=1}^u \xi_i z_{k-i} + \sum_{i=1}^w \zeta_i x_{k-i} + v_k \quad (2)$$

即

$$z_k = c_k + A z_{k-1} + H x_k + v_k \quad (3)$$

式中, z_k 为观测系统的输出(观测值), c_k 为观测系统的外生确定性输出项, A 为前定向量, H 为量测向量, x_k 为状态向量, v_k 为测量噪声。相关系数AR(p)、MA(q)和ARMA(p,q)序列可用状态方程统一表示为^[4]

$$f(z_n, z_{n-1}, \dots, z_1 | z_0) = \prod_{k=1}^n (2\pi P_{k|k-1}^z)^{-1/2} \exp \left[-\frac{(z_k - \hat{z}_{k|k-1})^2}{2P_{k|k-1}^z} \right] \quad (12)$$

两边取对数, 可得对数极大似然函数为

$$x_{k+1} = M_{k+1} + \Phi_{k+1,k} x_k + \epsilon_{k+1} \quad (4)$$

式中, M_{k+1} 为均值向量, $\Phi_{k+1,k}$ 为状态转移矩阵, ϵ_{k+1} 为状态噪声向量, 其协方差矩阵为 Θ_{k+1} 并且在 $k > i$ 时, $E(\epsilon_k \epsilon_i) = 0$ 和 $E(x_k \epsilon_i) = 0$; 对于任意 k 和 i , $E(\epsilon_k \epsilon_i) = 0$ 和 $E(x_k \epsilon_i) = 0$, 其中 0 为零向量(矩阵)。对于相关系数AR(p)、MA(q)和ARMA(p,q)模型, x_{k+1} , M_{k+1} , $\Phi_{k+1,k}$, ϵ_{k+1} 和 H 的具体含义可参见文献[4]。

2.2 参数估计

设测量噪声相互独立, 即

$$E(v_k v_i) = \begin{cases} R_k & k = i \\ 0 & k \neq i \end{cases} \quad (5)$$

此时, 有 $E(z_k v_i) = 0 (k < i)$ 。

可以证明^[5]: 如果 x_0 , ϵ 和 $v_k (k = 1, 2, \dots)$ 均服从正态分布, 则在 $z_{1-u}, z_{2-u}, \dots, z_{k-1}$ 已经测得的条件下, z_k 服从均值为 $\hat{z}_{k|k-1}$ 方差为 $P_{k|k-1}^z$ 的正态分布。其中 x_0 为状态初值, 对于相关系数AR(p)模型, $x_0 = (x_0, x_{-1}, \dots, x_{1-w^*})$; 对于相关系数MA(q)模型, $x_0 = (x_0, \dots, x_{-w}, \epsilon_0, \dots, \epsilon_{-q})$; 对于相关系数ARMA(p,q)模型, $x_0 = (x_0, \dots, x_{1-w^*}, \epsilon_0, \dots, \epsilon_{-q})$; 并且 $E(x_0 \epsilon_k) = 0 (k \geq 1)$, 其中 $w^* = \max\{w+1, p\}$ 。 $\hat{z}_{k|k-1}$ 为 z_k 关于观测值 $z_1, z_2, \dots, z_{k-1}$ 的最小均方误差估计, $P_{k|k-1}^z$ 为 $z_{k|k-1}$ 的均方误差。它们可以通过递推求得^[4]

$$\hat{z}_{k|k-1} = c_k + A \hat{z}_{k-1} + H \hat{x}_{k|k-1} \quad (6)$$

$$\hat{x}_{k|k-1} = \hat{M}_{k,k-1} + \Phi_{k,k-1} \hat{x}_{k-1|k-1} \quad (7)$$

$$\hat{x}_{k-1|k-1} = \hat{x}_{k-1|k-2} + P_{k-1|k-2}^x - 2H \times (P_{k-1|k-2}^z)^{-1} (z_{k-1} - \hat{z}_{k-1|k-2}) \quad (8)$$

$$P_{k|k-1}^z = H \hat{P}_{k|k-1}^x H + R_k \quad (9)$$

$$P_{k|k-1}^x = \Phi_{k,k-1} P_{k-1|k-1}^x \Phi_{k,k-1}' + \Theta_k \quad (10)$$

$$P_{k-1|k-1}^x = P_{k-1|k-2}^x - P_{k-1|k-2}^x H \times (P_{k-1|k-2}^z)^{-1} H \hat{P}_{k-1|k-2}^x \quad (11)$$

式中 $\hat{x}_{k|k-1}$ 和 $\hat{x}_{k-1|k-1}$ 分别为 x_k 和 x_{k-1} 关于观测值 $z_1, z_2, \dots, z_{k-1}$ 的最小均方误差估计, $P_{k|k-1}^x$ 和 $P_{k-1|k-1}^x$ 分别为 $x_{k|k-1}$ 和 $x_{k-1|k-1}$ 的均方误差。递推从 z_0 和 $x_0|_0$ 开始, 其中 $z_0 = (z_0, z_{-1}, \dots, z_{1-u})$ 为 $k = 0, -1, \dots, 1-u$ 时刻观测系统的输出; $x_0|_0$ 为 x_0 的无条件均值, 即 $x_0|_0 = E(x_0)$, 其均方误差为 $P_{x_0|_0}^x = \text{Cov}(x_0, x_0)$ 。

由此可知, 测量值 $z_1, z_2, \dots, z_n$ 的联合概率密度函数为

$$\ln L(a, b, \eta, \theta, \sigma_\epsilon) = -\frac{n}{2} \ln(2\pi) - \frac{1}{2} \sum_{k=1}^n \ln P_{k|k-1}^z - \frac{1}{2} \sum_{k=1}^n \frac{(z_k - \hat{z}_{k|k-1})^2}{P_{k|k-1}^z} \quad (13)$$

对上式求导, 得到求解参数 a, b, η, θ 和 σ_ϵ 的极大似然方程组

$$\frac{\partial \ln L(a, b, \eta, \theta, \sigma_\epsilon)}{\partial a_j} = \sum_{k=1}^n \frac{z_k - \hat{z}_{k|k-1}}{P_{k|k-1}^z} \frac{\partial \hat{z}_{k|k-1}}{\partial a_j} = 0 \quad j = 0, 1, \dots, r \quad (14)$$

$$\frac{\partial \ln L(a, b, \eta, \theta, \sigma_\epsilon)}{\partial b_j} = \sum_{k=1}^n \frac{z_k - \hat{z}_{k|k-1}}{P_{k|k-1}^z} \frac{\partial \hat{z}_{k|k-1}}{\partial b_j} + \frac{1}{2} \sum_{k=1}^n \left[\frac{(z_k - \hat{z}_{k|k-1})^2}{(P_{k|k-1}^z)^2} - \frac{1}{P_{k|k-1}^z} \right] \frac{\partial P_{k|k-1}^z}{\partial b_j} = 0 \quad j = 0, 1, \dots, s \quad (15)$$

$$\frac{\partial \ln L(a, b, \eta, \theta, \sigma_\epsilon)}{\partial \eta} = \sum_{k=1}^n \frac{z_k - \hat{z}_{k|k-1}}{P_{k|k-1}^z} \frac{\partial \hat{z}_{k|k-1}}{\partial \eta} + \frac{1}{2} \sum_{k=1}^n \left[\frac{(z_k - \hat{z}_{k|k-1})^2}{(P_{k|k-1}^z)^2} - \frac{1}{P_{k|k-1}^z} \right] \frac{\partial P_{k|k-1}^z}{\partial \eta} = 0 \quad j = 1, 2, \dots, p \quad (16)$$

$$\frac{\partial \ln L(a, b, \eta, \theta, \sigma_\epsilon)}{\partial \theta} = \sum_{k=1}^n \frac{z_k - \hat{z}_{k|k-1}}{P_{k|k-1}^z} \frac{\partial \hat{z}_{k|k-1}}{\partial \theta} + \frac{1}{2} \sum_{k=1}^n \left[\frac{(z_k - \hat{z}_{k|k-1})^2}{(P_{k|k-1}^z)^2} - \frac{1}{P_{k|k-1}^z} \right] \frac{\partial P_{k|k-1}^z}{\partial \theta} = 0 \quad j = 1, 2, \dots, q \quad (17)$$

$$\frac{\partial \ln L(a, b, \eta, \theta, \sigma_\epsilon)}{\partial \sigma_\epsilon} = \sum_{k=1}^n \frac{z_k - \hat{z}_{k|k-1}}{P_{k|k-1}^z} \frac{\partial \hat{z}_{k|k-1}}{\partial \sigma_\epsilon} + \frac{1}{2} \sum_{k=1}^n \left[\frac{(z_k - \hat{z}_{k|k-1})^2}{(P_{k|k-1}^z)^2} - \frac{1}{P_{k|k-1}^z} \right] \frac{\partial P_{k|k-1}^z}{\partial \sigma_\epsilon} = 0 \quad (18)$$

式中

$$\frac{\partial \hat{z}_{k|k-1}}{\partial a_j} = H \left(\frac{\partial M}{\partial a_j} + \Phi_{k,k-1} \frac{\partial \hat{x}_{k-1|k-1}}{\partial a_j} \right) \quad (19)$$

$$\frac{\partial \hat{x}_{k-1|k-1}}{\partial a_j} = \frac{\partial M_{k-1}}{\partial a_j} + \Phi_{k-1,k-2} \frac{\partial \hat{x}_{k-2|k-2}}{\partial a_j} - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-1} \frac{\partial \hat{z}_{k-1|k-2}}{\partial a_j} \quad (20)$$

$$\frac{\partial \hat{z}_{k|k-1}}{\partial b_j} = H \left(\frac{\partial M}{\partial b_j} + \frac{\partial \Phi_{k,k-1}}{\partial b_j} \hat{x}_{k-1|k-1} + \Phi_{k,k-1} \frac{\partial \hat{x}_{k-1|k-1}}{\partial b_j} \right) \quad (21)$$

$$\frac{\partial \hat{x}_{k-1|k-1}}{\partial b_j} = \frac{\partial M_{k-1}}{\partial b_j} + \frac{\partial \Phi_{k-1,k-2}}{\partial b_j} \hat{x}_{k-2|k-2} + \Phi_{k-1,k-2} \frac{\partial \hat{x}_{k-2|k-2}}{\partial b_j} + \frac{\partial P_{k-1|k-2}^x}{\partial b_j} H (P_{k-1|k-2}^z)^{-1} (z_{k-1} - \hat{z}_{k-1|k-2}) - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-2} \frac{\partial \hat{z}_{k-1|k-2}}{\partial b_j} (z_{k-1} - \hat{z}_{k-1|k-2}) - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-1} \frac{\partial \hat{z}_{k-1|k-2}}{\partial b_j} \quad (22)$$

$$\frac{\partial P_{k|k-1}^z}{\partial b_j} = H \frac{\partial P_{k|k-1}^x}{\partial b_j} H \quad (23)$$

$$\frac{\partial P_{k|k-1}^x}{\partial b_j} = \frac{\partial \Phi_{k,k-1}}{\partial b_j} P_{k-1|k-1}^x \Phi_{k,k-1}' + \Phi_{k,k-1} \frac{\partial P_{k-1|k-1}^x}{\partial b_j} \Phi_{k,k-1}' + \Phi_{k,k-1} P_{k-1|k-1}^x \frac{\partial \Phi_{k,k-1}'}{\partial b_j} + \frac{\partial \Theta}{\partial b_j} \quad (24)$$

$$\frac{\partial P_{k-1|k-1}^x}{\partial b_j} = \frac{\partial P_{k-1|k-2}^x}{\partial b_j} - \frac{\partial P_{k-1|k-2}^x}{\partial b_j} H (P_{k-1|k-2}^z)^{-1} H P_{k-1|k-2}^x + P_{k-1|k-2}^x H \times (P_{k-1|k-2}^z)^{-2} \frac{\partial \hat{z}_{k-1|k-2}}{\partial b_j} H P_{k-1|k-2}^x - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-1} H \frac{\partial P_{k-1|k-2}^x}{\partial b_j} \quad (25)$$

$$\frac{\partial \hat{z}_{k|k-1}}{\partial \eta} = H \left(\frac{\partial M}{\partial \eta} + \frac{\partial \Phi_{k,k-1}}{\partial \eta} \hat{x}_{k-1|k-1} + \Phi_{k,k-1} \frac{\partial \hat{x}_{k-1|k-1}}{\partial \eta} \right) \quad (26)$$

$$\frac{\partial \hat{x}_{k-1|k-1}}{\partial \eta} = \frac{\partial M_{k-1}}{\partial \eta} + \frac{\partial \Phi_{k-1,k-2}}{\partial \eta} \hat{x}_{k-2|k-2} + \Phi_{k-1,k-2} \frac{\partial \hat{x}_{k-2|k-2}}{\partial \eta} + \frac{\partial P_{k-1|k-2}^x}{\partial \eta} H (P_{k-1|k-2}^z)^{-1} (z_{k-1} - \hat{z}_{k-1|k-2}) - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-2} \frac{\partial \hat{z}_{k-1|k-2}}{\partial \eta} (z_{k-1} - \hat{z}_{k-1|k-2}) - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-1} \frac{\partial \hat{z}_{k-1|k-2}}{\partial \eta} \quad (27)$$

$$\frac{\partial P_{k|k-1}^z}{\partial \eta} = H \frac{\partial P_{k|k-1}^x}{\partial \eta} H \quad (28)$$

$$\frac{\partial P_{k|k-1}^x}{\partial \eta} = \frac{\partial \Phi_{k,k-1}}{\partial \eta} P_{k-1|k-1}^x \Phi_{k,k-1}' + \Phi_{k,k-1} \frac{\partial P_{k-1|k-1}^x}{\partial \eta} \Phi_{k,k-1}' + \Phi_{k,k-1} P_{k-1|k-1}^x \frac{\partial \Phi_{k,k-1}'}{\partial \eta} \quad (29)$$

$$\frac{\partial P_{k-1|k-1}^x}{\partial \eta} = \frac{\partial P_{k-1|k-2}^x}{\partial \eta} - \frac{\partial P_{k-1|k-2}^x}{\partial \eta} H (P_{k-1|k-2}^z)^{-1} H P_{k-1|k-2}^x + P_{k-1|k-2}^x H \times$$

$$(P_{k-1|k-2}^z)^{-2} \frac{\partial P_{k-1|k-2}^z}{\partial \eta} H \hat{p}_{k-1|k-2}^x - P_{k-1|k-2}^x (P_{k-1|k-2}^z)^{-1} H \frac{\partial \hat{p}_{k-1|k-2}^x}{\partial \eta} \quad (30)$$

$$\frac{\partial \hat{x}_{k|k-1}}{\partial \theta} = H \left[\frac{\partial \Phi_{k,k-1}}{\partial \theta} \hat{x}_{k-1|k-1} + \Phi_{k,k-1} \frac{\partial \hat{x}_{k-1|k-1}}{\partial \theta} \right] \quad (31)$$

$$\frac{\partial \hat{x}_{k-1|k-1}}{\partial \theta} = \frac{\partial \Phi_{k-1,k-2}}{\partial \theta} \hat{x}_{k-2|k-2} + \Phi_{k-1,k-2} \frac{\partial \hat{x}_{k-2|k-2}}{\partial \theta} + \frac{\partial P_{k-1|k-2}^x}{\partial \theta} H (P_{k-1|k-2}^z)^{-1} (z_{k-1} - \hat{z}_{k-1|k-2}) - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-2} \frac{\partial P_{k-1|k-2}^z}{\partial \theta} (z_{k-1} - \hat{z}_{k-1|k-2}) - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-1} \frac{\partial \hat{z}_{k-1|k-2}}{\partial \theta} \quad (32)$$

$$\frac{\partial \hat{p}_{k|k-1}^x}{\partial \theta} = H \frac{\partial \hat{p}_{k-1|k-1}^x}{\partial \theta} H \quad (33)$$

$$\frac{\partial \hat{p}_{k|k-1}^x}{\partial \theta} = \frac{\partial \Phi_{k,k-1}}{\partial \theta} P_{k-1|k-1}^x \Phi_{k,k-1}' + \Phi_{k,k-1} \frac{\partial P_{k-1|k-1}^x}{\partial \theta} \Phi_{k,k-1}' + \Phi_{k,k-1} P_{k-1|k-1}^x \frac{\partial \Phi_{k,k-1}'}{\partial \theta} \quad (34)$$

$$\frac{\partial \hat{p}_{k-1|k-1}^x}{\partial \theta} = \frac{\partial \hat{p}_{k-1|k-2}^x}{\partial \theta} - \frac{\partial P_{k-1|k-2}^x}{\partial \theta} H (P_{k-1|k-2}^z)^{-1} H \hat{p}_{k-1|k-2}^x + P_{k-1|k-2}^x H \times (P_{k-1|k-2}^z)^{-2} \frac{\partial P_{k-1|k-2}^z}{\partial \theta} H \hat{p}_{k-1|k-2}^x - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-1} H \frac{\partial \hat{p}_{k-1|k-2}^x}{\partial \theta} \quad (35)$$

$$\frac{\partial \hat{z}_{k|k-1}}{\partial \theta} = H \Phi_{k,k-1} \frac{\partial \hat{z}_{k-1|k-1}}{\partial \theta} \quad (36)$$

$$\frac{\partial \hat{z}_{k-1|k-1}}{\partial \theta} = \Phi_{k-1,k-2} \frac{\partial \hat{z}_{k-2|k-2}}{\partial \theta} + \frac{\partial P_{k-1|k-2}^x}{\partial \theta} H (P_{k-1|k-2}^z)^{-1} (z_{k-1} - \hat{z}_{k-1|k-2}) - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-2} \frac{\partial P_{k-1|k-2}^z}{\partial \theta} (z_{k-1} - \hat{z}_{k-1|k-2}) - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-1} \frac{\partial \hat{z}_{k-1|k-2}}{\partial \theta} \quad (37)$$

$$\frac{\partial \hat{p}_{k|k-1}^x}{\partial \theta} = H \frac{\partial \hat{p}_{k-1|k-1}^x}{\partial \theta} H \quad (38)$$

$$\frac{\partial \hat{p}_{k|k-1}^x}{\partial \theta} = \Phi_{k,k-1} \frac{\partial \hat{p}_{k-1|k-1}^x}{\partial \theta} \Phi_{k,k-1}' + \frac{\partial \Phi_{k,k-1}}{\partial \theta} \quad (39)$$

$$\frac{\partial \hat{p}_{k-1|k-1}^x}{\partial \theta} = \frac{\partial \hat{p}_{k-1|k-2}^x}{\partial \theta} - \frac{\partial P_{k-1|k-2}^x}{\partial \theta} H (P_{k-1|k-2}^z)^{-1} H \hat{p}_{k-1|k-2}^x + P_{k-1|k-2}^x H \times (P_{k-1|k-2}^z)^{-2} \frac{\partial P_{k-1|k-2}^z}{\partial \theta} H \hat{p}_{k-1|k-2}^x - P_{k-1|k-2}^x H (P_{k-1|k-2}^z)^{-1} H \frac{\partial \hat{p}_{k-1|k-2}^x}{\partial \theta} \quad (40)$$

对上述非线性方程组进行数值求解时,若采用拟牛顿迭代法、梯度法等传统计算方法求解该非线性方程组,则往往容易发散或收敛很慢,甚至导致不同的初值,收敛于不同的点。所以,建议采用文献[6]中方法求解。

当数据总量 n 较大时,可令 $z_0 = \mathbf{0}$ 和 $x_0 = E(x_0)$, 此时有 $P_{0|0}^x = \mathbf{0}$ 。此外,当测量噪声相关时,则可采用文献[4]中方法,先将其转化为测量噪声相互独立的情况,然后再用上述方法处理。

3 系统参数识别

在上面的讨论中观测系统的外生确定性输出项 c_k 、前定向量 A 、量测向量 H 、测量噪声 v_k 的方差 R_k 均为已知,但是,工程实际中有时这些系统参数是未知的,需要对其进行识别。设观测系统的外生确定性输出项 c_k 可表示为

$$c_k = a_{c0} \mathcal{Q}_0(t_k) + a_{c1} \mathcal{Q}_1(t_k) + \cdots + a_{c_{r_c}} \mathcal{Q}_{r_c}(t_k) \quad (41)$$

测量噪声 v_k 的方差 R_k 可表示为

$$R_k = I^2(b_v, t_k) \alpha_0^2 = [1 + b_{v1} \phi_{v1}(t_k) + \cdots + b_{v_{s_v}} \phi_{v_{s_v}}(t_k)] \alpha_0^2 \quad (42)$$

其中, $a_c = (a_{c0}, a_{c1}, \cdots, a_{c_{r_c}})'$, $b_v = (b_{v1}, b_{v2}, \cdots, b_{v_{s_v}})'$ 为待定参数; $\mathcal{Q}_0(t_k) = 1$; $\mathcal{Q}_i(t_k)$, $i = 1, 2, \cdots, r_c$ 和 $\phi_i(t_k)$, $i = 1, 2, \cdots, s_v$ 均为已知函数。

3.1 测量噪声相互独立情况下的系统参数识别

当系统的输入 $x_{1-w}, x_{2-w}, \cdots, x_n$ 及系统的输出 $z_{1-u}, z_{2-u}, \cdots, z_n$ 已知时,可以利用极大似然估计方法对系统参数 $a_c, b_v, \xi = (\xi_1, \xi_2, \cdots, \xi_r)'$, $\zeta = (\zeta_1, \zeta_2, \cdots, \zeta_s)$ 和 α_0 进行估计。当测量噪声相互独立时,根据系统输出 $z_1, z_2, \cdots, z_n$ 的联合概率密度,可得系统参数的对数似然函数为

$$\ln L(a_c, b_v, \xi, \zeta, \sigma_0) = -\frac{n}{2} \ln(2\pi) - \sum_{k=1}^n \ln[I(b_v, t_k) \sigma_0] - \frac{1}{2} \sum_{k=1}^n \frac{1}{I^2(b_v, t_k) \sigma_0^2} [z_k - \sum_{i=0}^{r_c} a_{ci} \varphi_{ci}(t_k) - \sum_{i=1}^u \xi_{iZ_{k-i}} - \sum_{j=1}^w \zeta_{jX_{k-j}}]^2 \quad (43)$$

对上式求偏导, 得到求解参数 a_c, ξ, ζ, b_v 和 σ_0 的方程组:

$$\omega = (a_c', \xi', \zeta')' = L_{\omega\omega}^{-1} L_{\omega 0} \omega_0 \quad (44)$$

$$\sigma_0^2 = \frac{1}{n} \sum_{k=1}^n (z_k - L_{\omega\omega}^{-1} L_{\omega 0} \omega_0)^2 I^{-2}(b_v, t_k) \quad (45)$$

$$\Delta_j(b_v) = \sigma_0^2 l_\phi + l_z \phi + L_{\omega\omega}^{-1} L_{\omega\omega j} L_{\omega\omega}^{-1} L_{\omega 0} \omega_0 - 2 L_{\omega\omega}^{-1} L_{\omega\omega j} L_{\omega\omega}^{-1} L_{\omega 0} \omega_0 = 0 \quad j = 1, 2, \dots, s_v \quad (46)$$

式中 $L_{\omega\omega} = (l_{ij})_{m \times m}$, $L_{\omega 0} = (l_{0i})_{m \times 1}$, $L_{\omega\omega j} = (l_{ij})_{m \times m}$, $L_{\omega 0 j} = (l_{0i})_{m \times 1}$, $\beta_k = (\beta_{ik})_{m \times 1}$, 其中 $m = r_c + u + w + 2$, 并且有

$$l_{ii} = \sum_{k=1}^n \beta_{ik} \beta_{ik} I^{-2}(b_v, t_k) \quad (47)$$

$$l_{0i} = \sum_{k=1}^n \beta_{ik} z_k I^{-2}(b_v, t_k) \quad (48)$$

$$l_{ii}^* = \sum_{k=1}^n \beta_{ik} \beta_{ik} \phi_j(t_k) I^{-3}(b_v, t_k) \quad (49)$$

$$l_{0i}^* = \sum_{k=1}^n \beta_{ik} z_k \phi_j(t_k) I^{-3}(b_v, t_k) \quad (50)$$

$$\beta_{ik} = \begin{cases} \varphi_{(i-1)}(t_k) & 1 \leq i \leq r_c + 1 \\ z_{k+r_c-i+1} & r_c + 2 \leq i \leq r_c + u + 1 \\ x_{k+r_c+u-i+2} & r_c + u + 2 \leq i \leq m \end{cases} \quad (51)$$

$$l_\phi = - \sum_{k=1}^n \frac{\phi_j(t_k)}{I(b_v, t_k)} \quad (52)$$

$$l_z \phi = \sum_{k=1}^n \frac{z_k^2 \phi_j(t_k)}{I^3(b_v, t_k)} \quad (53)$$

对于上述方程组, 可先采用文献[6]方法求解非线性方程组(46), 得到 b_v , 然后由式(44)和式(45)求得 a_c, ξ, ζ 和 σ_0

3.2 测量噪声相关情况下的系统参数识别

当测量噪声相关时, 由于 AR 模型与 MA 模型和 $ARMA$ 模型具有等效性^[7], 所以, 不妨设 $v_1, v_2, \dots, v_n$ 满足零均值相关系数 $AR(p_v)$ 模型(当测量噪声有趋势项时, 可将其趋势项归入到 c_k 中):

$$\frac{v_k}{I(b_v, t_k)} = \sum_{i=1}^{p_v} \frac{\eta_i v_{k-i}}{I(b_v, t_{k-i})} + v_k^* \quad (54)$$

式中 v_k^* 为白噪声序列, 其方差为 σ_v^2 。则观测方程可表示为

$$z_k^* = \sum_{i=0}^{r_c} a_{ci} \varphi_{ci}(t_k) + \sum_{i=1}^u \xi_{iZ_{k-i}} + \sum_{j=1}^w \zeta_{jX_{k-j}} + v_k^* \quad (55)$$

式中

$$z_k^* = \frac{z_k}{I(b_v, t_k)} - \sum_{i=1}^{p_v} \frac{\eta_i z_{k-i}}{I(b_v, t_{k-i})} \quad (56)$$

$$\varphi_{ci}^*(t_k) = \frac{\varphi_{ci}(t_k)}{I(b_v, t_k)} - \sum_{i=1}^{p_v} \frac{\eta_i \varphi_{ci}(t_{k-i})}{I(b_v, t_{k-i})} \quad (57)$$

$$x_{kj}^* = \frac{x_{kj}}{I(b_v, t_k)} - \sum_{i=1}^{p_v} \frac{\eta_i x_{k-i}}{I(b_v, t_{k-i})} \quad (58)$$

由于 v_k^* 相互独立, 所以根据 $z_1, z_2, \dots, z_n$ 的联合概率密度可知, 参数 $a_c, b_v, \xi, \zeta, \eta$ 和 σ_v^2 的对数似然函数为

$$\ln L(a_c, b_v, \xi, \zeta, \eta, \sigma_v^2) = -\frac{n}{2} \ln(2\pi) - n \ln \sigma_v^2 - \sum_{k=1}^n \ln I(b_v, t_k) - \frac{1}{2\sigma_v^2} \sum_{k=1}^n [z_k^* - \sum_{i=0}^{r_c} a_{ci} \varphi_{ci}^*(t_k) - \sum_{i=1}^u \xi_{iZ_{k-i}} - \sum_{j=1}^w \zeta_{jX_{k-j}}]^2 \quad (59)$$

对上式求偏导, 可得到求解参数 $a_c, b_v, \xi, \zeta, \eta$ 和 σ_v^2 的极大似然方程组

$$\frac{\partial \ln L(a_c, b_v, \xi, \zeta, \eta, \sigma_v^2)}{\partial a_{cj}} = \frac{1}{\sigma_v^2} \sum_{k=1}^n [z_k^* - \sum_{i=0}^{r_c} a_{ci} \varphi_{ci}^*(t_k) - \sum_{i=1}^u \xi_{iZ_{k-i}} - \sum_{j=1}^w \zeta_{jX_{k-j}}] \varphi_{cj}^*(t_k) = 0 \quad j = 0, 1, \dots, r_c \quad (60)$$

$$\frac{\partial \ln L(a_c, b_v, \xi, \zeta, \eta, \sigma_v^2)}{\partial \xi_j} = \frac{1}{\sigma_v^2} \sum_{k=1}^n [z_k^* - \sum_{i=0}^{r_c} a_{ci} \varphi_{ci}^*(t_k) - \sum_{i=1}^u \xi_{iZ_{k-i}} - \sum_{j=1}^w \zeta_{jX_{k-j}}] z_{k-j}^* = 0 \quad j = 1, 2, \dots, u \quad (61)$$

$$\frac{\partial \ln L(a_c, b_v, \xi, \zeta, \eta, \sigma_v^2)}{\partial \zeta_j} = \frac{1}{\sigma_v^2} \sum_{k=1}^n [z_k^* - \sum_{i=0}^{r_c} a_{ci} \varphi_{ci}^*(t_k) - \sum_{i=1}^u \xi_{iZ_{k-i}} - \sum_{j=1}^w \zeta_{jX_{k-j}}] x_{k-j}^* = 0 \quad j = 0, 1, \dots, w \quad (62)$$

$$\frac{\partial \ln L(a_c, b_v, \xi, \zeta, \eta, \sigma_v^2)}{\partial b_v} = -\frac{1}{\sigma_v^2} \sum_{k=1}^n [z_k^* - \sum_{i=0}^{r_c} a_{ci} \varphi_{ci}^*(t_k) - \sum_{i=1}^u \xi_{iZ_{k-i}} - \sum_{j=1}^w \zeta_{jX_{k-j}}] \times$$

$$\left[\frac{\partial z_k^*}{\partial b_{vj}} - \sum_{i=0}^{r_c} a_{ci} \frac{\partial \varphi_i(t_k)}{\partial b_{vj}} - \sum_{i=1}^u \xi_i \frac{\partial z_{k-i}^*}{\partial b_{vj}} - \sum_{i=0}^w \zeta_i \frac{\partial x_{k-i}^*}{\partial b_{vj}} \right] - \sum_{i=1}^n \frac{\phi_i(t_k)}{I(b_v, t_k)} = 0$$

$$j = 1, 2, \dots, s_v \quad (63)$$

$$\frac{\partial \ln L(a_c, b_v, \xi, \zeta, \eta, \alpha^*)}{\partial \eta_j} = \frac{1}{\sigma_v^*} \sum_{k=1}^n \left[z_k^* - \sum_{i=0}^{r_c} a_{ci} \varphi_i(t_k) - \sum_{i=1}^u \xi_i z_{k-i}^* - \sum_{i=0}^w \zeta_i x_{k-i}^* \right] \times$$

$$\left[\frac{z_{k-j}}{I(b_v, t_{k-j})} - \sum_{i=0}^{r_c} \frac{a_{ci} \varphi_i(t_{k-j})}{I(b_v, t_{k-j})} - \sum_{i=1}^u \frac{\xi_i z_{k-i-j}^*}{I(b_v, t_{k-i-j})} - \sum_{i=0}^w \frac{\zeta_i x_{k-i-j}^*}{I(b_v, t_{k-i-j})} \right] = 0$$

$$j = 0, 1, \dots, p_v \quad (64)$$

$$\frac{\partial \ln L(a_c, b_v, \xi, \zeta, \eta, \alpha^*)}{\partial \alpha^*} = -\frac{n}{\sigma_v^*} + \frac{1}{\sigma_v^*} \sum_{k=1}^n \left[z_k^* - \sum_{i=0}^{r_c} a_{ci} \varphi_i(t_k) - \sum_{i=1}^u \xi_i z_{k-i}^* - \sum_{i=0}^w \zeta_i x_{k-i}^* \right]^2 = 0 \quad (65)$$

式(65)可简化为

$$\alpha^* = \frac{1}{n} \sum_{k=1}^n \left[z_k^* - \sum_{i=0}^{r_c} a_{ci} \varphi_i(t_k) - \sum_{i=1}^u \xi_i z_{k-i}^* - \sum_{i=0}^w \zeta_i x_{k-i}^* \right]^2 \quad (66)$$

式中

$$\frac{\partial z_k^*}{\partial b_{vj}} = -\frac{z_k \phi_j(t_k)}{I^2(b_v, t_k)} + \sum_{i=1}^{p_v} \frac{\eta_i z_{k-i} \phi_j(t_{k-i})}{I^2(b_v, t_{k-i})} \quad (67)$$

$$\frac{\partial \varphi_i(t_k)}{\partial b_{vj}} = -\frac{\varphi_i(t_k) \phi_j(t_k)}{I^2(b_v, t_k)} + \sum_{i=1}^{p_v} \frac{\eta_i \varphi_i(t_{k-i}) \phi_j(t_{k-i})}{I^2(b_v, t_{k-i})} \quad (68)$$

$$\frac{\partial x_k^*}{\partial b_{vj}} = -\frac{x_k \phi_j(t_k)}{I^2(b_v, t_k)} + \sum_{i=1}^{p_v} \frac{\eta_i x_{k-i} \phi_j(t_{k-i})}{I^2(b_v, t_{k-i})} \quad (69)$$

4 结 论

(1) 针对相关系数平稳序列不可直接测得的情况,建立了通过观测数据估计相关系数 $AR(p)$ 、 $MA(q)$ 和 $ARMA(p, q)$ 模型参数的方法。

(2) 当测量噪声相互独立,其均值和方差随时间变化时,给出了观测系统参数识别的极大似然方法。

(3) 当测量噪声为相关系数 $AR(p)$ 序列时,建立了观测系统参数识别方法。当测量噪声为相关系数 $MA(q)$ 和 $ARMA(p, q)$ 序列时,可将其转化为相关系数 $AR(p)$ 序列处理。

(4) 由于考虑了测量噪声的均值和方差随时间变化情况,本文方法比传统方法更加符合工程实际,具有更高的估计和识别精度。

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