

文章编号: 1000-8055(2003)04-0470-07

非等间距相关系数平稳序列分析方法

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摘要: 工程实际中由于数据缺损或受测试条件所限, 常常得到非等间距的时间序列。而传统的时序分析方法只适用于等间距采样的数据, 目前工程上对这类问题通常采用插值等近似处理方法, 这往往导致较大误差。本文提出非等间距相关系数平稳序列的概念, 建立了非等间距相关系数平稳序列自回归模型, 给出了非等间距相关系数 $AR(p)$ 序列的精确极大似然估计和条件极大似然估计, 能够高精度地确定非等间距序列的均值函数、方差函数和相关系数函数。

关键词: 航空、航天推进系统; 时间序列; 相关系数平稳序列; 非等间距序列; 非平稳随机过程; 极大似然估计

中图分类号: TB301; O211 **文献标识码:** A

Analysis Method for Correlation Coefficient Stationary Series with Unequally Spaced Data

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Abstract: The idea of correlation coefficient stationary series with unequally spaced data is presented and the p th-order autoregressive model for the correlation coefficient stationary series with unequally spaced data is established. The exact maximum likelihood estimations and the conditional maximum likelihood estimations of their parameters are given in detail. Unequally spaced time series is common in engineering when the observation is irregular. The traditional time series analysis method is only suitable to equally spaced data and the approximate analysis methods for unequally spaced data, such as interpolation method, may cause big error. By employing the present method, the mean function, the variance function and the correlation coefficient function of unequally spaced time series can be calculated with high precision.

Key words: aerospace propulsion system; time series; correlation coefficient stationary series; unequally spaced series; non-stationary random processes; maximum likelihood estimation

收稿日期: 2003-05-20; **修订日期:** 2003-07-01

基金项目: 国防科技预研项目(413200204)

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1 引言

文献[1]提出相关系数平稳过程的概念, 从一般的非平稳随机过程中分离出一类工程上常见且便于研究的随机过程。文献[2~4]进一步提出相关系数 $AR(p)$, $MA(q)$ 和 $ARMA(p, q)$ 序列的概念, 并建立了相关系数 $AR(p)$, $MA(q)$ 和 $ARMA(p, q)$ 序列分析方法。工程上遇到的时间序列基本上属于相关系数平稳序列, 其均值和方差可随时间变化, 传统的均值和方差都为常数的相关函数平稳序列仅是它的一个特例。针对许多工程领域由于数据缺损或受测试条件所限, 常常得到非等间距时间序列的情况, 本文进一步建立非等间距相关系数平稳序列自回归模型, 给出非等间距相关系数 $AR(p)$ 序列的精确极大似然估计和条件极大似然估计。传统的时间序列分析方法只适用于等间距采样数据, 目前工程上通常采用插值等近似处理方法处理非等间距的时间序列, 这往往导致较大误差。本文方法则可以在时域进行全程分析, 具有信息量大的优点, 能够高精度地确定相关系数平稳序列的均值函数、方差函数和相关系数函数。从而解决了工程中非等间距时间序列分析处理的难题。

2 非等间距相关系数 $AR(1)$ 序列分析方法

设 $X(t)$ 为相关系数平稳过程, $X = (X_1, X_2, \dots, X_n)$ 为它的一个时间序列, 其中 $X_i = X(t_i)$, $i = 1, 2, \dots, n$, 那么根据文献[2]可知, $Y(t) = [X(t) - \mu(a, t)]/I(b, t)$ 为相关函数平稳过程, 若 $Y = (Y_1, Y_2, \dots, Y_n)$ 为 $AR(p)$, $MA(q)$ 或 $ARMA(p, q)$ 序列, 则称 $X = (X_1, X_2, \dots, X_n)$ 为相关系数 $AR(p)$, $MA(q)$ 或 $ARMA(p, q)$ 序列, 其中 $Y_i = [X(t_i) - \mu(a, t_i)]/I(b, t_i)$, $E[X(t)] = \mu(a, t)$, $\text{Var}[X(t)] = \sigma^2 I^2(b, t)$; $a = (a_0, a_1, \dots, a_r)'$, $b = (b_0, b_1, \dots, b_s)$ 和 σ_0 均为待定参数。当 $t_1, t_2, \dots, t_n$ 为非等间距时, 则称 $X = (X_1, X_2, \dots, X_n)$ 为非等间距相关系数平稳序列。

2.1 非等间距相关系数 $AR(1)$ 模型

根据一阶连续系统自回归模型^[5]和文献[2]可知, 相关系数 $AR(1)$ 序列 $X = (X_1, X_2, \dots, X_n)$ 的相关系数函数为:

$$\rho(\tau) = e^{\alpha\tau} \quad (1)$$

式中 τ 为时间间隔。自回归系数由下式给出:

$$\eta(\tau) = \rho(\tau) \quad (2)$$

因此, 非等间距相关系数 $AR(1)$ 模型为:

$$Y_i = e^{\alpha\tau_i} Y_{i-1} + \epsilon(\tau_i) \quad (3)$$

式中 $Y_i = [X(t_i) - \mu(a, t_i)]/I(b, t_i)$, $\tau_i = t_i - t_{i-1}$, $\epsilon(\tau_i)$ 是相互独立且服从从 $N(0, \sigma_\epsilon^2)$ 的随机变量。其中:

$$\sigma_\epsilon^2 = \sigma_0^2(1 - e^{2\alpha\tau_i}) \quad (4)$$

2.2 非等间距相关系数 $AR(1)$ 序列精确极大似然估计

设 $x_{ji} = x(t_{ji})$, $i = 1, 2, \dots, n_j$ 是来自相关系数平稳正态过程 $X(t)$ 的第 j 个样本序列的一组观测值, 若 $x_j = (x_{j1}, x_{j2}, \dots, x_{jn_j})$ 为非等间距相关系数 $AR(1)$ 序列, 则 $y_j = (y_{j1}, y_{j2}, \dots, y_{jn_j})$ 为非等间距相关函数 $AR(1)$ 序列, 其中 $y_{ji} = [x_{ji} - \mu(a, t_{ji})]/I(b, t_{ji})$, $j = 1, 2, \dots, m$ 。由 $x_1, x_2, \dots, x_m$ 的联合密度函数^[2]可知, 参数 a, b 和 α, σ_0 的精确似然函数为:

$$L(a, b, \alpha, \sigma_0) = \prod_{j=1}^m \left[\frac{1}{\sqrt{2\pi\sigma_0 I(b, t_{j1})}} \exp\left(-\frac{y_{j1}^2}{2\sigma_0^2}\right) \times \prod_{i=2}^{n_j} \frac{\exp\left[-\frac{(y_{ji} - e^{\alpha\tau_{ji}} y_{j(i-1)})^2}{2\sigma_0^2 \phi(\alpha, \tau_{ji})}\right]}{\sqrt{2\pi\phi(\alpha, \tau_{ji}) \sigma_0 I(b, t_{ji})}} \right] \quad (5)$$

式中:

$$\phi(\alpha, \tau_{ji}) = 1 - e^{2\alpha\tau_{ji}} \quad (6)$$

对式(5)两边取对数, 得精确对数似然函数为:

$$\ln L(a, b, \alpha, \sigma_0) = \sum_{j=1}^m \left[-\frac{n_j}{2} \ln(2\pi\sigma_0^2) - \frac{y_{j1}^2}{2\sigma_0^2} - \frac{1}{2} \sum_{i=2}^{n_j} \ln \phi(\alpha, \tau_{ji}) - \frac{1}{2} \sum_{i=2}^{n_j} \frac{(y_{ji} - e^{\alpha\tau_{ji}} y_{j(i-1)})^2}{\sigma_0^2 \phi(\alpha, \tau_{ji})} - \sum_{i=2}^{n_j} \ln I(b, t_{ji}) \right] \quad (7)$$

对式(7)求偏导, 可得求解 a, b 和 α, σ_0 的精确极大似然方程组:

$$\frac{\partial \ln L(a, b, \alpha, \sigma_0)}{\partial a_k} = -\frac{1}{\sigma_0^2} \sum_{j=1}^m \left[y_{j1} \frac{\partial y_{j1}}{\partial a_k} + \sum_{i=2}^{n_j} \frac{y_{ji} - e^{\alpha\tau_{ji}} y_{j(i-1)}}{\phi(\alpha, \tau_{ji})} \left(\frac{\partial y_{ji}}{\partial a_k} - e^{\alpha\tau_{ji}} \frac{\partial y_{j(i-1)}}{\partial a_k} \right) \right] = 0 \quad (8)$$

$$\frac{\partial \ln L(a, b, \alpha, \sigma_0)}{\partial b_k} = -\frac{1}{\sigma_0^2} \sum_{j=1}^m \left[y_{j1} \frac{\partial y_{j1}}{\partial b_k} + \sum_{i=2}^{n_j} \frac{y_{ji} - e^{\alpha\tau_{ji}} y_{j(i-1)}}{\phi(\alpha, \tau_{ji})} \left(\frac{\partial y_{ji}}{\partial b_k} - e^{\alpha\tau_{ji}} \frac{\partial y_{j(i-1)}}{\partial b_k} \right) \right] -$$

$$\sum_{j=1}^m \sum_{i=2}^{n_j} \frac{1}{I(b, t_{ji})} \frac{\partial I(b, t_{ji})}{\partial b_k} = 0 \quad (9)$$

$$k = 0, 1, \dots, s$$

$$\frac{\partial \ln L(a, b, \alpha, \sigma_0)}{\partial \alpha} = - \sum_{j=1}^m \sum_{i=1}^{n_j} \frac{1}{2\phi(\alpha, \tau_{ji})} \frac{\partial \phi(\alpha, \tau_{ji})}{\partial \alpha} + \frac{1}{\sigma_0^2} \sum_{j=1}^m \sum_{i=1}^{n_j} \left[\frac{e^{\alpha \tau_{ji}} (y_{ji} - \frac{e^{\alpha \tau_{ji}} y_{j(i-1)}}{\phi(\alpha, \tau_{ji})}) \tau_{ji} y_{j(i-1)}}{2\phi(\alpha, \tau_{ji})} + \frac{(y_{ji} - \frac{e^{\alpha \tau_{ji}} y_{j(i-1)}}{\phi(\alpha, \tau_{ji})})^2}{2\phi(\alpha, \tau_{ji})} \frac{\partial \phi(\alpha, \tau_{ji})}{\partial \alpha} \right] = 0 \quad (10)$$

$$\frac{\partial \ln L(a, b, \alpha, \sigma_0)}{\partial \sigma_0} = - \sum_{j=1}^m \left[\frac{n_j}{\sigma_0} - \frac{y_{j1}^2}{\sigma_0^3} - \frac{1}{\sigma_0^3} \sum_{i=1}^{n_j} \frac{(y_{ji} - \frac{e^{\alpha \tau_{ji}} y_{j(i-1)}}{\phi(\alpha, \tau_{ji})})^2}{\phi(\alpha, \tau_{ji})} \right] = 0 \quad (11)$$

式中:

$$\frac{\partial \phi(\alpha, \tau_{ji})}{\partial \alpha} = -2\tau_{ji} e^{2\alpha \tau_{ji}} \quad (12)$$

式(11)进一步简化为:

$$\sigma_0^2 = \frac{1}{n} \sum_{j=1}^m \left[y_{j1}^2 + \sum_{i=1}^{n_j} \frac{(y_{ji} - \frac{e^{\alpha \tau_{ji}} y_{j(i-1)}}{\phi(\alpha, \tau_{ji})})^2}{\phi(\alpha, \tau_{ji})} \right] \quad (13)$$

式中:

$$n = \sum_{j=1}^m n_j \quad (14)$$

a, b 和 α, σ_0 由式(8)~式(11)联立求解。非线性方程组(8)~式(10)需迭代计算,若采用拟牛顿迭代法、梯度法等传统计算方法,则往往容易发散或收敛很慢,甚至导致不同的初值,收敛于不同的结果。所以,建议采用文献[6]中方法求解。

$$\phi(\alpha, \tau_i) = \frac{1 - \rho_{i1}^2 - \rho_{i2}^2 - \rho_{i(i-1)}^2 + 2\rho_{i1}\rho_{i2}\rho_{i(i-1)}}{1 - \rho_{i(i-1)}^2} \quad (21)$$

3.2 非等间距相关系数 $AR(2)$ 序列精确极大似然估计

由文献[2]可知, a, b, α 和 σ_0 的精确对数似然函数为:

$$\ln L(a, b, \alpha, \sigma_0) = \sum_{j=1}^m \left[-\frac{n_j}{2} \ln(2\pi\sigma_0^2) - \frac{1}{2} \ln(1 - \rho_{j21}^2) - \frac{1}{2\sigma_0^2} \frac{y_{j1}^2 + y_{j2}^2 - 2y_{j1}y_{j2}\rho_{j21}}{1 - \rho_{j21}^2} - \frac{1}{2} \sum_{i=1}^{n_j} \ln \phi(\alpha, \tau_{ji}) - \frac{1}{2} \sum_{i=1}^{n_j} \frac{[y_{ji} - \frac{\eta(\tau_{ji})y_{j(i-1)}}{\phi(\alpha, \tau_{ji})} - \frac{\eta_k(\tau_{ji})y_{j(i-2)}}{\phi(\alpha, \tau_{ji})}]^2}{\phi(\alpha, \tau_{ji})} - \sum_{i=1}^{n_j} \ln I(b, t_{ji}) \right] \quad (22)$$

对式(22)求偏导,得到 a, b, α 和 σ_0 的精确极大似然方程组:

$$\frac{\partial \ln L(a, b, \alpha, \sigma_0)}{\partial a_k} = - \frac{1}{\sigma_0^2} \sum_{j=1}^m \left\{ \frac{1}{1 - \rho_{j21}^2} \left[y_{j1} \frac{\partial y_{j1}}{\partial a_k} + y_{j2} \frac{\partial y_{j2}}{\partial a_k} - \rho_{j21} \left(y_{j1} \frac{\partial y_{j2}}{\partial a_k} + y_{j2} \frac{\partial y_{j1}}{\partial a_k} \right) \right] + \sum_{i=1}^{n_j} \frac{y_{ji} - \frac{\eta(\tau_{ji})y_{j(i-1)}}{\phi(\alpha, \tau_{ji})} - \frac{\eta_k(\tau_{ji})y_{j(i-2)}}{\phi(\alpha, \tau_{ji})}}{\phi(\alpha, \tau_{ji})} \left[\frac{\partial y_{ji}}{\partial a_k} - \eta(\tau_{ji}) \frac{\partial y_{j(i-1)}}{\partial a_k} - \eta_k(\tau_{ji}) \frac{\partial y_{j(i-2)}}{\partial a_k} \right] \right\} = 0 \quad (23)$$

$k = 0, 1, \dots, r$

$$\frac{\partial \ln L(a, b, \alpha, \sigma_0)}{\partial b_k} = - \frac{1}{\sigma_0^2} \sum_{j=1}^m \left\{ \frac{1}{1 - \rho_{j21}^2} \left[y_{j1} \frac{\partial y_{j1}}{\partial b_k} + y_{j2} \frac{\partial y_{j2}}{\partial b_k} - \rho_{j21} \left(y_{j1} \frac{\partial y_{j2}}{\partial b_k} + y_{j2} \frac{\partial y_{j1}}{\partial b_k} \right) \right] + \sum_{i=1}^{n_j} \frac{y_{ji} - \frac{\eta(\tau_{ji})y_{j(i-1)}}{\phi(\alpha, \tau_{ji})} - \frac{\eta_k(\tau_{ji})y_{j(i-2)}}{\phi(\alpha, \tau_{ji})}}{\phi(\alpha, \tau_{ji})} \left[\frac{\partial y_{ji}}{\partial b_k} - \eta(\tau_{ji}) \frac{\partial y_{j(i-1)}}{\partial b_k} - \eta_k(\tau_{ji}) \frac{\partial y_{j(i-2)}}{\partial b_k} \right] \right\} - \sum_{j=1}^m \sum_{i=1}^{n_j} \frac{1}{I(b, t_{ji})} \frac{\partial I(b, t_{ji})}{\partial b_k} = 0 \quad k = 0, 1, \dots, s \quad (24)$$

3 非等间距相关系数 $AR(2)$ 序列分析方法

3.1 非等间距相关系数 $AR(2)$ 模型

同理,根据二阶连续系统自回归模型^[5]和文献[2]可知,相关系数 $AR(2)$ 序列 $X = (X_1, X_2, \dots, X_n)$ 的相关系数函数为:

$$\rho(\tau) = \frac{1}{\alpha_2 - \alpha_1} (\alpha_2 e^{\alpha_1 \tau} - \alpha_1 e^{\alpha_2 \tau}) \quad (15)$$

式中 τ 为时间间隔。由尤尔-瓦尔克方程可求得自回归系数:

$$\eta_1(\tau_i) = \frac{\rho_{i1} - \rho_{i2}\rho_{i(i-1)}}{1 - \rho_{i(i-1)}^2} \quad (16)$$

$$\eta_2(\tau_i) = \frac{\rho_{i2} - \rho_{i1}\rho_{i(i-1)}}{1 - \rho_{i(i-1)}^2} \quad (17)$$

式中:

$$\rho_k = \frac{1}{\alpha_2 - \alpha_1} (\alpha_2 e^{\alpha_1 \tau_k} - \alpha_1 e^{\alpha_2 \tau_k}) \quad k = 1, 2 \quad (18)$$

$$\rho_{i(i-1)} = \frac{1}{\alpha_2 - \alpha_1} (\alpha_2 e^{\alpha_1 \tau_{i(i-1)}} - \alpha_1 e^{\alpha_2 \tau_{i(i-1)}}) \quad (19)$$

因此,非等间距相关系数 $AR(2)$ 模型为:

$$Y_i = \eta_1(\tau_i) Y_{i-1} + \eta_2(\tau_i) Y_{i-2} + \epsilon(\tau_i) \quad (20)$$

式中 $\tau_i = (\tau_{i1}, \tau_{i2})'$, $\tau_{i1} = t_i - t_{i-1}$, $\tau_{i2} = t_i - t_{i-2}$, $\epsilon(\tau_i)$ 相互独立且服从 $N[0, \sigma_0^2 \phi(\alpha, \tau_i)]$, 其中 $\alpha = (\alpha_1, \alpha_2)'$, $\phi(\alpha, \tau_i)$ 由下式给出:

$$\begin{aligned} \frac{\partial \ln L(a, b, \alpha, \alpha_k)}{\partial \alpha_k} = & \sum_{j=1}^m \left[\frac{\rho_{j21}}{1 - \rho_{j21}^2} \frac{\partial \rho_{j21}}{\partial \alpha_k} - \sum_{i=3}^{n_j} \frac{1}{2\phi(\alpha, \tau_{ji})} \frac{\partial \phi(\alpha, \tau_{ji})}{\partial \alpha_k} \right] + \\ & \frac{1}{\alpha_0^2} \sum_{j=1}^m \left\{ \frac{y_{j1}y_{j2}}{1 - \rho_{j21}^2} \frac{\partial \rho_{j21}}{\partial \alpha_k} - \frac{\rho_{j21}(y_{j1}^2 + y_{j2}^2 - 2y_{j1}y_{j2}\rho_{j21})}{(1 - \rho_{j21}^2)^2} \frac{\partial \rho_{j21}}{\partial \alpha_k} + \right. \\ & \sum_{i=3}^{n_j} \left[\frac{y_{ji} - \frac{\eta(\tau_{ji})y_{j(i-1)} - \eta_k(\tau_{ji})y_{j(i-2)}}{\phi(\alpha, \tau_{ji})} \left(\frac{\partial \eta(\tau_{ji})}{\partial \alpha_k} y_{j(i-1)} + \frac{\partial \eta_k(\tau_{ji})}{\partial \alpha_k} y_{j(i-2)} \right) + \right. \\ & \left. \left. \frac{[y_{ji} - \frac{\eta(\tau_{ji})y_{j(i-1)} - \eta_k(\tau_{ji})y_{j(i-2)}}{2\phi(\alpha, \tau_{ji})}]^2}{2\phi(\alpha, \tau_{ji})} \frac{\partial \phi(\alpha, \tau_{ji})}{\partial \alpha_k} \right] \right\} = 0 \quad k = 1, 2 \end{aligned} \quad (25)$$

$$\frac{\partial \ln L(a, b, \alpha, \alpha_k)}{\partial \alpha_0} =$$

$$- \sum_{j=1}^m \left\{ \frac{n_j}{\alpha_0} - \frac{1}{\alpha_0^3} \frac{y_{j1}^2 + y_{j2}^2 - 2y_{j1}y_{j2}\rho_{j21}}{1 - \rho_{j21}^2} - \frac{1}{\alpha_0^3} \sum_{i=3}^{n_j} \frac{[y_{ji} - \frac{\eta(\tau_{ji})y_{j(i-1)} - \eta_k(\tau_{ji})y_{j(i-2)}}{\phi(\alpha, \tau_{ji})}]^2}{\phi(\alpha, \tau_{ji})} \right\} = 0 \quad (26)$$

上式简化为:

$$\alpha_0 = \frac{1}{n} \sum_{j=1}^m \left\{ \frac{y_{j1}^2 + y_{j2}^2 - 2y_{j1}y_{j2}\rho_{j21}}{1 - \rho_{j21}^2} + \sum_{i=3}^{n_j} \frac{[y_{ji} - \frac{\eta(\tau_{ji})y_{j(i-1)} - \eta_k(\tau_{ji})y_{j(i-2)}}{\phi(\alpha, \tau_{ji})}]^2}{\phi(\alpha, \tau_{ji})} \right\} \quad (27)$$

式(25)中:

$$\frac{\partial \eta(\tau_{ji})}{\partial \alpha_k} = \frac{\partial \eta(\tau_{ji})}{\partial \rho_{j11}} \frac{\partial \rho_{j11}}{\partial \alpha_k} + \frac{\partial \eta(\tau_{ji})}{\partial \rho_{j12}} \frac{\partial \rho_{j12}}{\partial \alpha_k} + \frac{\partial \eta(\tau_{ji})}{\partial \rho_{j(i-1)1}} \frac{\partial \rho_{j(i-1)1}}{\partial \alpha_k} \quad k = 1, 2 \quad (28)$$

$$\frac{\partial \eta_k(\tau_{ji})}{\partial \alpha_k} = \frac{\partial \eta_k(\tau_{ji})}{\partial \rho_{j11}} \frac{\partial \rho_{j11}}{\partial \alpha_k} + \frac{\partial \eta_k(\tau_{ji})}{\partial \rho_{j12}} \frac{\partial \rho_{j12}}{\partial \alpha_k} + \frac{\partial \eta_k(\tau_{ji})}{\partial \rho_{j(i-1)1}} \frac{\partial \rho_{j(i-1)1}}{\partial \alpha_k} \quad k = 1, 2 \quad (29)$$

$$\frac{\partial \phi(\alpha, \tau_{ji})}{\partial \alpha_k} = \frac{\partial \phi(\alpha, \tau_{ji})}{\partial \rho_{j11}} \frac{\partial \rho_{j11}}{\partial \alpha_k} + \frac{\partial \phi(\alpha, \tau_{ji})}{\partial \rho_{j12}} \frac{\partial \rho_{j12}}{\partial \alpha_k} + \frac{\partial \phi(\alpha, \tau_{ji})}{\partial \rho_{j(i-1)1}} \frac{\partial \rho_{j(i-1)1}}{\partial \alpha_k} \quad k = 1, 2 \quad (30)$$

其中:

$$\frac{\partial \phi(\alpha, \tau_{ji})}{\partial \rho_{j11}} = \frac{2(\rho_{j11} - \rho_{j12}\rho_{j(i-1)1})}{\rho_{j(i-1)1}^2 - 1} \quad (31)$$

$$\frac{\partial \phi(\alpha, \tau_{ji})}{\partial \rho_{j12}} = \frac{2(\rho_{j12} - \rho_{j11}\rho_{j(i-1)1})}{\rho_{j(i-1)1}^2 - 1} \quad (32)$$

$$\frac{\partial \phi(\alpha, \tau_{ji})}{\partial \rho_{j(i-1)1}} = \frac{2(\rho_{j12} - \rho_{j11}\rho_{j(i-1)1})(\rho_{j11} - \rho_{j12}\rho_{j(i-1)1})}{(\rho_{j(i-1)1}^2 - 1)^2} \quad (33)$$

$$\frac{\partial \eta_{ik}}{\partial \alpha_k} = \frac{\alpha_k [e^{\alpha_1 \tau_{jk}} - e^{\alpha_2 \tau_{jk}} - \tau_{jk} e^{\alpha_1 \tau_{jk}} (\alpha_k - \alpha_k)]}{(\alpha_k - \alpha_k)^2} \quad k = 1, 2 \quad (34)$$

$$\frac{\partial \eta_{ik}}{\partial \alpha_k} = \frac{\alpha_k [e^{\alpha_2 \tau_{jk}} - e^{\alpha_1 \tau_{jk}} + \tau_{jk} e^{\alpha_2 \tau_{jk}} (\alpha_k - \alpha_k)]}{(\alpha_k - \alpha_k)^2} \quad k = 1, 2 \quad (35)$$

$$\frac{\partial \eta(\tau_{ji})}{\partial \rho_{j11}} = \frac{1}{1 - \rho_{j(i-1)1}^2} \quad (36)$$

$$\frac{\partial \eta(\tau_{ji})}{\partial \rho_{j12}} = \frac{\rho_{j(i-1)1}}{\rho_{j(i-1)1}^2 - 1} \quad (37)$$

$$\frac{\partial \eta(\tau_{ji})}{\partial \rho_{j(i-1)1}} = \frac{2\rho_{j11}\rho_{j(i-1)1} - \rho_{j12}(1 + \rho_{j(i-1)1}^2)}{(\rho_{j(i-1)1}^2 - 1)^2} \quad (38)$$

$$\frac{\partial \eta_k(\tau_{ji})}{\partial \rho_{j11}} = \frac{\rho_{j(i-1)1}}{\rho_{j(i-1)1}^2 - 1} \quad (39)$$

$$\frac{\partial \eta_k(\tau_{ji})}{\partial \rho_{j12}} = \frac{1}{1 - \rho_{j(i-1)1}^2} \quad (40)$$

$$\frac{\partial \eta_k(\tau_{ji})}{\partial \rho_{j(i-1)1}} = \frac{2\rho_{j12}\rho_{j(i-1)1} - \rho_{j11}(1 + \rho_{j(i-1)1}^2)}{(\rho_{j(i-1)1}^2 - 1)^2} \quad (41)$$

4 非等间距相关系数 $AR(p)$ 序列分析方法

4.1 非等间距相关系数 $AR(p)$ 模型

一般的, 非等间距相关系数 $AR(p)$ 模型具有如下形式:

$$Y(t_i) = \sum_{k=1}^p \eta_k(\tau_i) Y(t_{i-k}) + \epsilon(\tau_i) \quad (42)$$

式中 $\tau_i = (\tau_{i1}, \tau_{i2}, \dots, \tau_{ip})'$, $\tau_{ik} = t_i - t_{i-k}$ ($k = 1, 2, \dots, p$)。 $\epsilon(\tau_i)$ 相互独立且服从 $N[0, \sigma^2 \phi(\alpha, \tau_i)]$, 其中 $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_p)'$, $\phi(\alpha, \tau_i)$ 由下式给出:

$$\phi(\alpha, \tau_i) =$$

$$1 - 2 \sum_{l=1}^p \eta_l(\tau_i) \rho_{il} + \sum_{l,s=1}^p \eta_l(\tau_i) \eta_s(\tau_i) \rho_{l|l-s|} \quad (43)$$

式中 $l \leq i - \min(l, s)$ 。自回归系数 $\eta = \{\eta_1(\tau_i), \eta_2(\tau_i), \dots, \eta_p(\tau_i)\}$ 可由尤尔-瓦耳克方程求得:

$$\eta = V_i^{-1} \rho_i \quad (44)$$

式中:

$$V_i = \begin{bmatrix} 1 & \rho_{(i-1)1} & \rho_{(i-1)2} & \cdots & \rho_{(i-1)(p-1)} \\ \rho_{(i-1)1} & 1 & \rho_{(i-2)1} & \cdots & \rho_{(i-2)(p-2)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \rho_{(i-1)(p-1)} & \rho_{(i-2)(p-2)} & \cdots & \cdots & 1 \end{bmatrix} \quad (45)$$

其中:

$$\rho_{uw} = \frac{\sum_{i=1}^p \beta_i e^{\alpha_i T_{uw}}}{\sum_{i=1}^p \beta_i} \quad (46)$$

$$\beta_i = \lambda \sum_{j=1}^p \frac{\lambda_j}{\alpha_i + \alpha_j} \quad (47)$$

$$\lambda = \frac{1}{\prod_{j=1, j \neq i}^p (\alpha_i - \alpha_j)} \quad (48)$$

向量 $\rho_i = (\rho_{i1}, \rho_{i2}, \cdots, \rho_{ip})'$ 。相关系数函数 ρ_{uw} 由下式给出^[2, 5]:

4.2 非等间距相关系数 $AR(p)$ 序列精确极大似然估计

由文献[2]可知, a, b, α 和 σ_0 的精确对数似然函数为:

$$\ln L(a, b, \alpha, \sigma_0) = \sum_{j=1}^m \left\{ -\frac{n_j}{2} \ln(2\pi\sigma_0^2) + \frac{1}{2} \ln |V_{j(p+1)}^{-1}| - \frac{1}{2\sigma_0^2} y'_{jp} V_{j(p+1)}^{-1} y_{jp} - \frac{1}{2} \sum_{i=p+1}^{n_j} \ln \phi(\alpha, \tau_{ji}) - \frac{1}{2} \sum_{i=p+1}^{n_j} \frac{[y_{ji} - \sum_{k=1}^p \eta(\tau_{ji}) y_{j(i-k)}]^2}{\sigma_0^2 \phi(\alpha, \tau_{ji})} - \sum_{i=1}^{n_j} \ln I(b, t_{ji}) \right\} \quad (49)$$

式中 $y_{jp} = (y_{j1}, y_{j2}, \cdots, y_{jp})'$; $V_{j(p+1)}$ 为 y_{jp} 的相关系数矩阵, 可由式(45)求得。对式(49)求偏导, 得到 a, b, α 和 σ_0 的精确极大似然方程组:

$$\frac{\partial \ln L(a, b, \alpha, \sigma_0)}{\partial a_k} = -\frac{1}{\sigma_0^2} \sum_{j=1}^m \left\{ y'_{jp} V_{j(p+1)}^{-1} \frac{\partial y_{jp}}{\partial a_k} + \sum_{i=p+1}^{n_j} \frac{y_{ji} - \sum_{k=1}^p \eta(\tau_{ji}) y_{j(i-k)}}{\phi(\alpha, \tau_{ji})} \left[\frac{\partial y_{ji}}{\partial a_k} - \sum_{k=1}^p \eta(\tau_{ji}) \frac{\partial y_{j(i-k)}}{\partial a_k} \right] \right\} = 0 \quad k = 0, 1, \cdots, r \quad (50)$$

$$\frac{\partial \ln L(a, b, \alpha, \sigma_0)}{\partial b_k} = -\frac{1}{\sigma_0^2} \sum_{j=1}^m \left\{ y'_{jp} V_{j(p+1)}^{-1} \frac{\partial y_{jp}}{\partial b_k} + \sum_{i=p+1}^{n_j} \frac{y_{ji} - \sum_{k=1}^p \eta(\tau_{ji}) y_{j(i-k)}}{\phi(\alpha, \tau_{ji})} \left[\frac{\partial y_{ji}}{\partial b_k} - \sum_{k=1}^p \eta(\tau_{ji}) \frac{\partial y_{j(i-k)}}{\partial b_k} \right] \right\} - \sum_{j=1}^m \sum_{i=1}^{n_j} \frac{1}{I(b, t_{ji})} \frac{\partial I(b, t_{ji})}{\partial b_k} = 0 \quad k = 0, 1, \cdots, s \quad (51)$$

$$\frac{\partial \ln L(a, b, \alpha, \sigma_0)}{\partial \alpha_k} = \sum_{j=1}^m \left[\frac{1}{2 |V_{j(p+1)}^{-1}|} \frac{\partial |V_{j(p+1)}^{-1}|}{\partial \alpha_k} - \sum_{i=p+1}^{n_j} \frac{1}{2 \phi(\alpha, \tau_{ji})} \frac{\partial \phi(\alpha, \tau_{ji})}{\partial \alpha_k} \right] - \frac{1}{\sigma_0^2} \sum_{j=1}^m \left\{ \frac{1}{2} y'_{jp} \frac{\partial V_{j(p+1)}^{-1}}{\partial \alpha_k} y_{jp} - \sum_{i=p+1}^{n_j} \left[\frac{y_{ji} - \sum_{k=1}^p \eta(\tau_{ji}) y_{j(i-k)}}{\phi(\alpha, \tau_{ji})} \sum_{k=1}^p \frac{\partial \eta(\tau_{ji})}{\partial \alpha_k} y_{j(i-k)} + \frac{[y_{ji} - \sum_{k=1}^p \eta(\tau_{ji}) y_{j(i-k)}]^2}{2 \phi(\alpha, \tau_{ji})} \frac{\partial \phi(\alpha, \tau_{ji})}{\partial \alpha_k} \right] \right\} = 0 \quad k = 1, 2, \cdots, p \quad (52)$$

$$\frac{\partial \ln L(a, b, \alpha, \sigma_0)}{\partial \sigma_0} = -\sum_{j=1}^m \left\{ \frac{n_j}{\sigma_0} - \frac{1}{\sigma_0^3} y'_{jp} V_{j(p+1)}^{-1} y_{jp} - \frac{1}{\sigma_0^3} \sum_{i=p+1}^{n_j} \frac{[y_{ji} - \sum_{k=1}^p \eta(\tau_{ji}) y_{j(i-k)}]^2}{\phi(\alpha, \tau_{ji})} \right\} = 0 \quad (53)$$

上式可简化为:

$$\sigma_0^2 = \frac{1}{n} \sum_{j=1}^m \left\{ y'_{jp} V_{j(p+1)}^{-1} y_{jp} + \sum_{i=p+1}^{n_j} \frac{[y_{ji} - \sum_{k=1}^p \eta(\tau_{ji}) y_{j(i-k)}]^2}{\phi(\alpha, \tau_{ji})} \right\} \quad (54)$$

式(52)中:

$$\frac{\partial V_{j(p+1)}^{-1}}{\partial \alpha_k} = -V_{j(p+1)}^{-1} \frac{\partial V_{j(p+1)}}{\partial \alpha_k} V_{j(p+1)}^{-1} \quad (55)$$

$$\frac{\partial |V_{j(p+1)}|}{\partial \alpha_k} = - \frac{1}{|V_{j(p+1)}|^2} \sum_{u,w=1}^p V_{juw} \frac{\partial V_{juw}}{\partial \alpha_k} \quad (56)$$

式中 V_{juw} 为矩阵 $V_{j(p+1)}$ 中第 u 行, 第 w 列元素, V_{juw} 为元素 V_{juw} 的代数余子式。

4.3 非等间距相关系数 $A_R(p)$ 序列条件极大似然估计

根据精确似然函数进行估计, 往往较为复杂。当样本较大时, 采用条件似然函数既简单又能很好的满足实际要求。下面将给出非等间距相关系数 $A_R(p)$ 序列条件极大似然估计。根据文献[2]可知, 在每个样本序列的前 p 个观测值已知的条件下 a, b, α 和 σ 的条件对数似然函数为:

$$\ln L(a, b, \alpha, \sigma) = \sum_{j=1}^m \left\{ - \frac{n_j - p}{2} \ln(2\pi\sigma^2) - \frac{1}{2} \sum_{i=p+1}^{n_j} \ln \phi(\alpha, \tau_{ji}) - \frac{1}{2} \sum_{i=p+1}^{n_j} \frac{[y_{ji} - \sum_{k=1}^p \eta_k(\tau_{ji}) y_{j(i-k)}]^2}{\sigma^2 \phi(\alpha, \tau_{ji})} - \sum_{i=p+1}^{n_j} \ln I(b, t_{ji}) \right\} \quad (57)$$

对式(57)求偏导, 得到 a, b, α 和 σ 的条件极大似然方程组:

$$\frac{\partial \ln L(a, b, \alpha, \sigma)}{\partial \alpha_k} = - \frac{1}{\sigma^2} \sum_{j=1}^m \left\{ \sum_{i=p+1}^{n_j} \frac{y_{ji} - \sum_{k=1}^p \eta_k(\tau_{ji}) y_{j(i-k)}}{\phi(\alpha, \tau_{ji})} \left[\frac{\partial y_{ji}}{\partial \alpha_k} - \sum_{k=1}^p \eta_k(\tau_{ji}) \frac{\partial y_{j(i-k)}}{\partial \alpha_k} \right] \right\} = 0 \quad k = 0, 1, \dots, r \quad (58)$$

$$\frac{\partial \ln L(a, b, \alpha, \sigma)}{\partial b_k} = - \frac{1}{\sigma^2} \sum_{j=1}^m \left\{ \sum_{i=p+1}^{n_j} \frac{y_{ji} - \sum_{k=1}^p \eta_k(\tau_{ji}) y_{j(i-k)}}{\phi(\alpha, \tau_{ji})} \left[\frac{\partial y_{ji}}{\partial b_k} - \sum_{k=1}^p \eta_k(\tau_{ji}) \frac{\partial y_{j(i-k)}}{\partial b_k} \right] \right\} - \sum_{j=1}^m \sum_{i=p+1}^{n_j} \frac{1}{I(b, t_{ji})} \frac{\partial I(b, t_{ji})}{\partial b_k} = 0 \quad k = 0, 1, \dots, s \quad (59)$$

$$\frac{\partial \ln L(a, b, \alpha, \sigma)}{\partial \alpha_k} = - \sum_{j=1}^m \sum_{i=p+1}^{n_j} \frac{1}{2 \phi(\alpha, \tau_{ji})} \frac{\partial \phi(\alpha, \tau_{ji})}{\partial \alpha_k} + \frac{1}{\sigma^2} \sum_{j=1}^m \sum_{i=p+1}^{n_j} \left\{ \frac{y_{ji} - \sum_{k=1}^p \eta_k(\tau_{ji}) y_{j(i-k)}}{\phi(\alpha, \tau_{ji})} \sum_{k=1}^p \frac{\partial \eta_k(\tau_{ji})}{\partial \alpha_k} y_{j(i-k)} + \frac{[y_{ji} - \sum_{k=1}^p \eta_k(\tau_{ji}) y_{j(i-k)}]^2}{2 \phi(\alpha, \tau_{ji})} \frac{\partial \phi(\alpha, \tau_{ji})}{\partial \alpha_k} \right\} = 0 \quad k = 1, 2, \dots, p \quad (60)$$

$$\frac{\partial \ln L(a, b, \alpha, \sigma)}{\partial \sigma} = - \sum_{j=1}^m \left\{ \frac{n_j - p}{\sigma} - \frac{1}{\sigma^3} \sum_{i=p+1}^{n_j} \frac{[y_{ji} - \sum_{k=1}^p \eta_k(\tau_{ji}) y_{j(i-k)}]^2}{\phi(\alpha, \tau_{ji})} \right\} = 0 \quad (61)$$

上式可简化为:

$$\sigma^2 = \frac{1}{n - pm} \sum_{j=1}^m \sum_{i=p+1}^{n_j} \frac{[y_{ji} - \sum_{k=1}^p \eta_k(\tau_{ji}) y_{j(i-k)}]^2}{\phi(\alpha, \tau_{ji})} \quad (62)$$

式(52)、(60)中:

$$\frac{\partial \phi(\alpha, \tau_{ji})}{\partial \alpha_k} = - 2 \sum_{l=1}^p \left(\frac{\partial \eta_l(\tau_{ji})}{\partial \alpha_k} \rho_{jil} + \eta_l(\tau_{ji}) \frac{\partial \rho_{jil}}{\partial \alpha_k} \right) + \sum_{l,s=1}^p \left(\frac{\partial \eta_l(\tau_{ji})}{\partial \alpha_k} \eta_s(\tau_{ji}) \rho_{jil} \eta_{l-s} + \eta_l(\tau_{ji}) \frac{\partial \eta_s(\tau_{ji})}{\partial \alpha_k} \rho_{jil} \eta_{l-s} + \eta_l(\tau_{ji}) \eta_s(\tau_{ji}) \frac{\partial \rho_{jil} \eta_{l-s}}{\partial \alpha_k} \right) \quad (63)$$

$$\frac{\partial \eta_l}{\partial \alpha_k} = V_l^{-1} \left(\frac{\partial \rho_l}{\partial \alpha_k} - \frac{\partial V_l}{\partial \alpha_k} V_l^{-1} \rho_l \right) \quad (64)$$

对于矩阵 V_l 及向量 ρ_l 中元素 ρ_{luv} 求偏导得:

$$\frac{\partial \rho_{luv}}{\partial \alpha_k} = \frac{\left(\sum_{i=1}^p \frac{\partial \beta_i}{\partial \alpha_k} e^{\alpha_i \tau_{luv}} + \beta_k \tau_{luv} e^{\alpha_k \tau_{luv}} \right) \sum_{i=1}^p \beta_i - \sum_{i=1}^p \beta_i e^{\alpha_i \tau_{luv}} \sum_{i=1}^p \frac{\partial \beta_i}{\partial \alpha_k}}{\left(\sum_{i=1}^p \beta_i \right)^2} \quad (65)$$

$$\frac{\partial \beta_i}{\partial \alpha_k} = \frac{\partial \lambda}{\partial \alpha_k} \sum_{j=1}^p \frac{\lambda_j}{\alpha_j + \alpha_i} + \lambda_i \sum_{j=1}^p \frac{\frac{\partial \lambda_j}{\partial \alpha_k} (\alpha_j + \alpha_i) - \lambda_j (\delta_{jk} + \delta_{ik})}{(\alpha_j + \alpha_i)^2} \quad (66)$$

上式中 δ_{ik} 和 δ_{jk} 为 Kronecker 函数。即当 $i = k$ 时, $\delta_{ik} = 1$; 当 $i \neq k$ 时, $\delta_{ik} = 0$ 。 δ_{jk} 与之同理。

$$\frac{\partial \lambda}{\partial \alpha_k} = \begin{cases} - \sum_{l \neq i}^p \frac{\lambda_l}{\alpha_l - \alpha_i} & i = k \\ \frac{\lambda_i}{\alpha_i - \alpha_k} & i \neq k \end{cases} \quad (67)$$

5 结 论

(1) 提出非等间距相关系数平稳序列的概念, 建立了非等间距相关系数平稳序列自回归模型。传统的时间序列分析方法只适用于等间距采样数据, 目前工程上通常采用插值等近似处理方法处理非等间距的时间序列, 这往往导致较大误差。本文方法则能够高精度地确定非等间距序列的均值函数、方差函数和相关系数函数。

(2) 详细讨论了在工程中有广泛应用的一阶和二阶非等间距相关系数平稳序列自回归模型, 分别建立了其精确极大似然方程。它可以在时域进行全程分析, 具有信息量大的优点。

(3) 建立了非等间距相关系数 $AR(p)$ 序列的精确极大似然估计和条件极大似然估计, 前者在样本较小时仍可满足高精度的要求; 后者则在样本较大时具有相当高的精度, 且简单便于工程应用。

(4) 对于滑动平均模型和自回归滑动平均模型, 在满足可逆性条件时, 可先将其转化为自回归模型, 然后采用本文方法进行处理。

(5) 采用文献[2]中方法可以对非等间距相关系数平稳序列进行频谱分析。

(6) 在性能测试中, 本文方法只需测定一个样本函数就能够对产品性能的均值和方差进行估计, 与传统的需要测定一组样本函数的成组试验法相比, 在精度相同的情况下可以节省大量试验费用和时间。

参考文献

- [1] 傅惠民. 相关系数平稳过程方法[J]. 机械强度, 2002, 24(3): 400—404
- [2] 傅惠民, 刘成瑞, 马小兵. 相关系数平稳过程时频分析方法[J]. 航空动力学报, 2002, 17(5): 504—512
- [3] 傅惠民, 马小兵. 相关系数 $MA(q)$ 序列分析方法[J]. 机械强度, 2003, 25(2): 183—186
- [4] 傅惠民, 刘成瑞. 相关系数 $ARMA(p, q)$ 序列分析方法[J]. 航空动力学报, 2003, 18(3): 161—166
- [5] 杨叔子, 吴雅等. 时间序列分析的工程应用(上册)[M]. 武汉: 华中理工大学出版社, 1991.
- [6] 傅惠民, 张应福, 张少波. 解非线性方程组的一元化方法[J]. 机械强度, 1999, 21(3): 205—207.
- [7] (美)詹姆斯D. 汉密尔顿. 时间序列分析[M]. 刘明志译. 北京: 中国社会科学出版社, 1999.
- [8] Robinson, P.M. Estimation of a Time Series Model from Unequally Spaced Data[J]. Stochastic Processes and their Applications, 1977, 6: 9—24
- [9] 刘次华. 随机过程[M]. 武汉: 华中科技大学出版社, 2001.