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频域内振动路径的功率流传递度排序

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摘 要: 基于振动的功率流理论和一般概率摄动法, 研究了多源激励浮筏隔振系统频域内振动传递路径的功率流传递概率的度量问题, 提出了频域内振动传递路径系统的功率流传递度的新概念和方法, 在考虑工程中的不确定因素以后, 在频域内清晰地描述了振动传递路径系统的功率流传递度。
关 键 词: 浮筏; 功率流; 传递路径; 传递概率; 传递度
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Powerflow transfer ratio of vibration path systems in frequency range

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Abstract: Based on the theory of powerflow and the generalized probabilistic perturbation method, this paper studied the path powerflow transfer probability of multiple sources exciting floating raft system in frequency range; and, the new concept and effective approach of path powerflow transfer ratio for vibration transfer path systems in frequency range were presented. The powerflow transfer ratio among multiple and/ or multi-dimensional paths was described correctly and expressly in frequency range when uncertain factors were considered.

Key words: floating raft; powerflow; transfer path; transfer probability; transfer ratio

在振动系统中, 振源、传递路径和接受结构之间相互作用, 传递路径作为承上启下的子系统是研究的重点. 如何清晰正确地阐明多重/ 维路径的贡献量是个重要的问题^[1-3]. Knuth 和 Singh^[3] 用介入损失的指标定量地评价路径的贡献度. 功率流理论常被用来分析振动问题, 是分析振动传递路径的有效方法, 国内外大量学者^[4-10] 对功率流理论有深入的研究研究.

本文基于振动功率流理论和一般概率摄动法^[11-14], 研究了多激励浮筏隔振传递路径系统的功率流和功率流传递率. 给出了传递路径系统的功率流传递度的概念和算法, 得到了振动传递路

径系统的传递度曲线, 估计传递路径系统的敏感区域, 清晰的描述出了哪些路径对振动系统的贡献量较大, 为振动传递路径和功率流理论的研究提供了新的思路和方法.

1 传递路径系统的导纳功率流

多源激励浮筏隔振系统如图 1 所示, 图中有 n 个激励 $F_1, F_2, \dots, F_n$, 作用在质量为 m_i ($i=1, 2, \dots, n$) 的刚体上, 速度为 V_i ; 筏体为刚性的质量块 m_{n+1} ; 机器导纳记为 Y_{m_i} ; 隔振器导纳为 Y_{gi} ; 弹性基座导纳为 Y_b , 厚度为 h ; 这里 $i=1, 2, \dots, n$. 根据导纳功率流理论有

$$Y_{mi} = \frac{1}{j\omega m_i}, \quad (i = 1, 2, \dots, n+1) \quad (1)$$

$$Y_{gi} = \frac{1}{(k_i/j\omega) + c_i}, \quad (i = 1, 2, \dots, n) \quad (2)$$

$$Y_b = \frac{1}{8\sqrt{B\rho h}} \quad (3)$$

其中弯曲刚度 $B = \frac{h^3 E}{12(1-\delta^2)}$; 杨氏摩量 $E = 2.1 \times 10^{11} \text{ N} \cdot \text{m}^{-2}$; 泊松比 $\delta = 0.25$. 隔振器刚度和阻尼分别为 k_i 和 c_i ($i = 1, 2, \dots, n+1$). 下面以 $n=2$ (如图 2) 为例, 对浮阀隔振系统进行受力分析, 得到如下方程:

$$F_1 = j\omega m_1 V_1 + \frac{V_1 - V_3}{Y_{g1}} \quad (4a)$$

$$F_2 = j\omega m_2 V_2 + \frac{V_2 - V_3}{Y_{g2}} \quad (4b)$$

$$j\omega m_3 V_3 + \frac{V_3}{Y_{g3} + Y_b} = \frac{V_1 - V_3}{Y_{g1}} + \frac{V_2 - V_3}{Y_{g2}} \quad (4c)$$

解之可得 V_1, V_2, V_3 , 则系统路径 1, 2 传递到质量为 m_3 的浮筏功率流分别为

$$P_{Ti} = \frac{1}{2} \text{Re} \left[V_3^H \frac{V_i - V_3}{Y_{gi}} \right], \quad (i = 1, 2) \quad (5)$$

其中 V_3^H 为 V_3 的共轭复数. 系统路径 1、路径 2 的输入功率流分别为

$$P_{INi} = \frac{1}{2} \text{Re} [F_i V_i], \quad (i = 1, 2) \quad (6)$$

系统路径 1、路径 2 的功率流传递率分别为

$$T_i = P_{Ti}/P_{INi}, \quad (i = 1, 2) \quad (7)$$

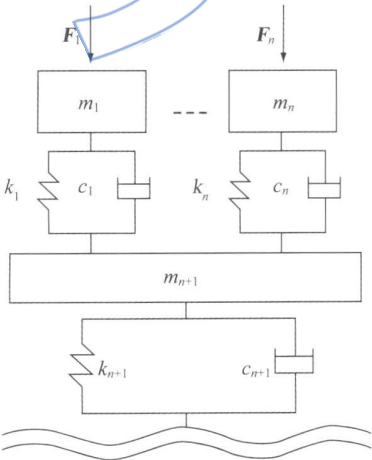


图 1 多源激励浮筏系统

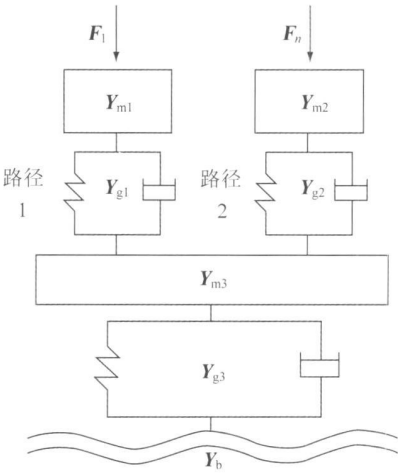


图 2 双激励浮筏系统

Fig. 2 Double sources exciting floating raft system

2 传递路径系统的传递度

路径参数的随机性, 设随机参数向量 $\mathbf{b} = [k_1 \ c_1 \ k_2 \ c_2]^T$ 代表路径的随机性, 服从正态分布. 因此必然导致服从正态分布的随机功率流 P_{Ti}, P_{INi} ($i = 1, 2$). 应用随机摄动技术, 把随机参数向量 $\mathbf{b}$ 、传递功率流和输入功率流 P_{Ti}, P_{INi} ($i = 1, 2$) 分解为均值与随机扰动两部分, 然后取数学期望和方差

$$\mathbf{b} = \mathbf{E}(\mathbf{b}) = \mathbf{E}(\mathbf{b}^d) + \mathbf{E}(\mathbf{b}^p) = \mathbf{b}^d \quad (8)$$

$$\mu_{P_{Ti}} = \mathbf{E}[P_{Ti}(\mathbf{b})] = \mathbf{E}[P_{Ti}^d(\mathbf{b})] +$$

$$\mathbf{E}[P_{Ti}^p(\mathbf{b})] = P_{Ti}^d(\mathbf{b}) \quad (9a)$$

$$\mu_{P_{INi}} = \mathbf{E}[P_{INi}(\mathbf{b})] = \mathbf{E}[P_{INi}^d(\mathbf{b})] +$$

$$\mathbf{E}[P_{INi}^p(\mathbf{b})] = P_{INi}^d(\mathbf{b}) \quad (9b)$$

$$\mathbf{\Phi} = \text{Var}(\mathbf{b}) =$$

$$\mathbf{E}\{[\mathbf{b} - \mathbf{E}(\mathbf{b})]^{[2]} \} = \mathbf{\Phi}^p \quad (10)$$

$$\sigma_{P_{Ti}}^2 = \text{Var}[P_{Ti}(\mathbf{b})] =$$

$$\mathbf{\Phi}^2 \mathbf{E} \left[\left(\frac{\partial P_{Ti}^d(\mathbf{b})}{\partial \mathbf{b}^T} \right)^{[2]} (\mathbf{b}^p)^{[2]} \right] = \left(\frac{\partial P_{Ti}^d(\mathbf{b})}{\partial \mathbf{b}^T} \right)^{[2]} \text{Var}(\mathbf{b}) \quad (11a)$$

$$\sigma_{P_{INi}}^2 = \text{Var}[P_{INi}(\mathbf{b})] =$$

$$\mathbf{\Phi}^2 \mathbf{E} \left[\left(\frac{\partial P_{INi}^d(\mathbf{b})}{\partial \mathbf{b}^T} \right)^{[2]} (\mathbf{b}^p)^{[2]} \right] = \left(\frac{\partial P_{INi}^d(\mathbf{b})}{\partial \mathbf{b}^T} \right)^{[2]} \text{Var}(\mathbf{b}) \quad (11b)$$

Fig. 1 Multiple sources exciting floating raft system

式中

$$\frac{\partial P_{Ti}^d}{\partial(\mathbf{b})^T} = \left[\frac{\partial P_{Ti}^d}{\partial k_1} \frac{\partial P_{Ti}^d}{\partial c_1} \frac{\partial P_{Ti}^d}{\partial k_2} \frac{\partial P_{Ti}^d}{\partial c_2} \right]$$
$$\frac{\partial P_{INi}^d}{\partial(\mathbf{b})^T} = \left[\frac{\partial P_{INi}^d}{\partial k_1} \frac{\partial P_{INi}^d}{\partial c_1} \frac{\partial P_{INi}^d}{\partial k_2} \frac{\partial P_{INi}^d}{\partial c_2} \right]$$

$E(\mathbf{b})$ 和 $\text{Var}(\mathbf{b})$ 分别代表基本随机变量的均值与方差矩阵; ε 为一小参数, 上标为 d 的部分表示随机变量中的确定部分, 上标为 p 的部分表示随机变量中的随机部分, 且具有零均值; $(\cdot)^{[2]} = (\cdot) \otimes (\cdot)$ 为 $(\cdot)$ 的二阶 Kronecker 幂, 符号 $\otimes$ 代表 Kronecker 积; P_{Ti}^p, P_{INi}^p 在 $E(P_{Ti}) = P_{Ti}^d, E(P_{INi}) = P_{INi}^d$ 附近用 Taylor 展开到一阶. 传递率的均值和方差表示为

$$\mu_r = E[\mathcal{N}_i^p] = \frac{E(P_{Ti})}{E(P_{INi})} \tag{12}$$

$$\sigma_r^2 = \text{Var}[\mathcal{N}_i^p] = \frac{[E(P_{Ti})]^2}{[E(P_{INi})]^2} \left\{ \frac{\text{Var}(P_{INi})}{[E(P_{INi})]^2} + \frac{\text{Var}(P_{Ti})}{[E(P_{Ti})]^2} - 2\rho \frac{\sqrt{\text{Var}(P_{INi})} \sqrt{\text{Var}(P_{Ti})}}{E(P_{INi}) E(P_{Ti})} \right\} \tag{13}$$

式中 ρ 为传递和输入功率流的相关系数. 传递系数定义为

$$\beta = \frac{\mu_r}{\sigma_r} = \frac{E[\mathcal{N}_i^p]}{\sqrt{\text{Var}[\mathcal{N}_i^p]}} \quad (i = 1, 2) \tag{14}$$

传递度定义为

$$R_{Ti} = \Phi(\beta) = \Phi\left(\frac{E[\mathcal{N}_i^p]}{\sqrt{\text{Var}[\mathcal{N}_i^p]}}\right) \quad (i = 1, 2) \tag{15}$$

式中 $\Phi(\cdot)$ 为标准正态分布函数.

3 数值算例

考虑如图 2 所示多源激励浮筏隔振系统, 对此系统的传递路径 1, 2 进行分析. 系统受到外力 $F_1 = 10^3 \text{ N}, F_2 = 1.3 \times 10^3 \text{ N}$ 激励, 路径的随机刚体质量 $m_1 = 50 \text{ kg}, m_2 = 77 \text{ kg}$, 筏体质量为 $m_3 = 220 \text{ kg}$, 无限大弹性基座厚度 $h = 0.03 \text{ m}$, 路径的随机质量、阻尼和刚度分别服从方差系数为 0.05 的正态分布, 隔振器的阻尼均值分别为 $c_1 = c_2 = c_3 = 0.15 \text{ N} \cdot \text{s/m}$; 的刚度均值分别为 $k_1 = 1.5 \times 10^4 \text{ N/m}, k_2 = 1.73 \times 10^4 \text{ N/m}, k_3 = 2.48 \times 10^5 \text{ N/m}$. 隔振器的阻尼均值分别为 $c_1 = c_2 = c_3 = 0.15 \text{ Ns/m}$.

1) 计算系统路径 1, 2 的输入和传递功率流 $P_{INi}, P_{Ti}, (i = 1, 2)$, 如图 3 和图 4 所示.

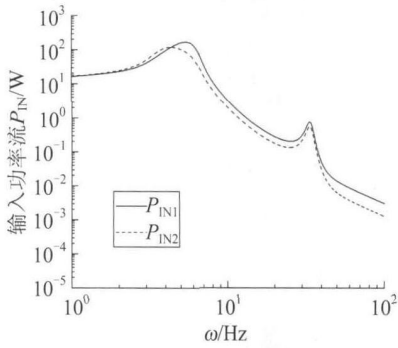


图 3 输入功率流关于激励频率的特性曲线
Fig. 3 Curve of imported power flow with regard to excitation frequency

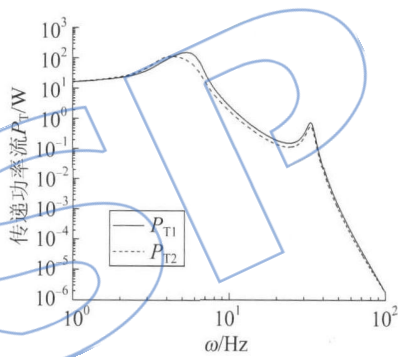


图 4 传递功率流关于激励频率的特性曲线
Fig. 4 Curve of transmitted power flow with regard to excitation frequency

从计算结果可以看出, 随着频率增大, 路径 1, 2 的输入功率流和传递功率流逐渐减小, 总体趋势相同. 路径 1 的输入功率流和传递功率流由略小于路径 2 逐渐变为略大于路径 2.

2) 计算路径 1, 2 的功率流传递率 λ_{Ti} 和传递度 $R_{Ti}, (i = 1, 2)$, 如图 5 和图 6 所示.

就此系统而言, 从计算结果图 5 可以看出, 功率流传递率在 0~1 范围内波动, 路径 1 的传递率由略大于路径 2, 逐渐变为略小于路径 2. 图 6 表示在有些频段传递度相对较高, 在这些频段上路径对接受结构振动的贡献量较大, 这主要决定于传递路径系统的参数特性. 图中曲线表示随着激励频率的增大路径 1 对接受结构振动的贡献量略小于路径 2.

3) 分别计算当 $k'_2 = 2.076 \times 10^4 \text{ N/m}$ 和 $c'_2 = 0.18 \text{ N} \cdot \text{s/m}$ 时路径 2 的功率流传递度 R_{T2}^k 和

R_{T2}^c , 如图 7 和图 8 所示.

就此系统而言, 从计算结果图 7 和图 8 可以看出, 在低频段传递路径系统的刚度对路径传递度影响较大, 在高频段传递路径系统的阻尼对路径传递度影响较大.

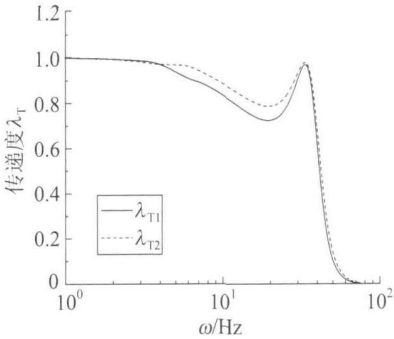


图 5 功率流传递率关于激励频率的特性曲线
Fig. 5 Curve of transmitted rate with regard to excitation frequency

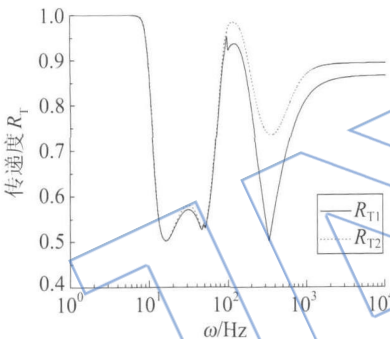


图 6 传递度关于激励频率的特性曲线
Fig. 6 Curve of transfer probability with regard to excitation frequency

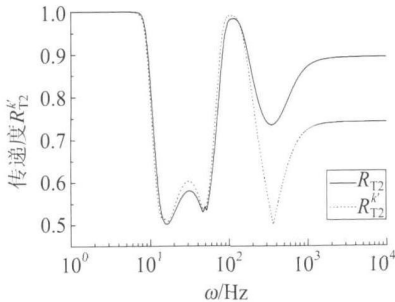


图 7 传递度 R_{T2}^c 关于激励频率的特性曲线
Fig. 7 Curve of transfer probability R_{T2}^c with regard to excitation frequency

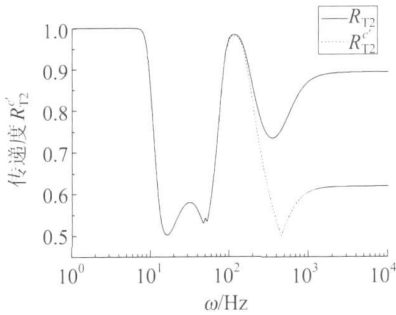


图 8 传递度 R_{T2}^c 关于激励频率的特性曲线
Fig. 8 Curve of transfer probability R_{T2}^c with regard to excitation frequency

4 结 论

本文提出了振动传递路径系统的功率流传递度的概念, 并获得了功率流传递度曲线, 分析了对传递路径系统传递度影响较大的随机参数, 研究了传递路径对接受结构振动的贡献量. 为工程实际中减振和隔振问题提供了新的分析方法, 为研究振动传递路径系统的路径功率流传递度量问题提供了新思路.

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