

自校准 Kalman 滤波方法

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摘 要: 提出一种自校准 Kalman 滤波方法(SKF), 建立 SKF 模型及其滤波递推算法. 在深空探测、发动机故障诊断等许多工程实际中, 由于未知输入(如突风、故障、未知的系统误差等)的影响, 传统的 Kalman 滤波方法在滤波递推过程中会产生较大误差. 文中提出的自校准 Kalman 滤波方法能够自动补偿这种未知输入的影响, 提高滤波精度. 从某飞行器仿真中可以看到, SKF 的滤波误差均值和方差分别比传统的 Kalman 滤波方法降低了 400% 和 300% 以上, 有效地改善了滤波效果. 并且该方法计算简单, 便于工程应用.

关 键 词: 自校准 Kalman 滤波; 未知输入; 滤波精度; 故障诊断; 深空探测

中图分类号: V448; O231

文献标志码: A

Self-calibration Kalman filter method

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Abstract: A self-calibration Kalman filter (SKF) method, whose model and recursive algorithm were established, was presented. In most practical cases, such as deep space exploration and engine fault diagnosis, because of the effect of unknown inputs, such as gust, fault and unknown system error, the well-known Kalman filter will lead to greater filtering error in recursive process. To solve this problem, the proposed SKF, which is applied to estimate and compensate the unknown inputs, efficiently reduces the effect of the unknown inputs and enhances filtering accuracy. For some spacecraft navigation simulation, the mean and variance of estimated state errors by SKF decreased by at least 400% and 300%, respectively. The SKF method can be effective to improve the performance of filter, simple to calculate and easy to apply in engineering.

Key words: self-calibration Kalman filter (SKF); unknown input; filtering accuracy; fault diagnosis; deep space exploration

自 1960 年 Kalman 提出 Kalman 滤波方法^[1]以来, 目前已广泛应用于信号处理、导航、深空探测和故障诊断等领域, 并不断地对 Kalman 滤波进行改进, 发展了一系列的滤波方法, 包括扩展 Kalman 滤波方法^[2]、无迹 Kalman 滤波方法^[3]、自适应 Kalman 滤波方法^[4]、粒子 Kalman 滤波方法^[5]等, 进一步提高 Kalman 滤波精度.

Kalman 滤波要求其状态方程和量测方程是精确的^[6-7], 但是, 在工程实际中, 由于环境因素的影响、测量设备的不稳定性、模型和参数的选取不当等原因, 状态方程或量测方程往往产生未知输入^[8-10], 这些未知输入直接降低了 Kalman 滤波精度. 如何修正这些未知输入的影响, 进一步提高 Kalman 滤波的精度是当前国际上研究的难点

收稿日期: 2014-02-29

基金项目: 国家重点基础研究发展计划(2012CB720000)

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和热点问题.为此,扩维 Kalman 滤波方法通过扩维将未知输入设计为状态变量增加在状态方程中进行滤波^[11-13].为了减少扩维带来的计算工作量,最优两步 Kalman 滤波^[14-15]和多步 Kalman 滤波^[16]也被用于解决这种未知输入问题,但这些方法都限于未知输入的变化规律可以被描述的情况.文献[17]介绍了一些自校准技术,文献[18-19]给出确定量测方程参数并对其进行验证、校准的方法.但是,自校准技术要求满足一些较为苛刻的条件,否则难以获得较好的效果^[17].

针对这一情况,通过深入系统的研究,本文提出一种自校准 Kalman 滤波方法,它能够自动补偿这种未知输入(如突风、故障、未知的系统误差等)的影响,从而降低滤波误差.该方法不需要事先已知未知输入任何信息,也不需要增加新的外测设备和太多的计算工作量,就可以提高 Kalman 滤波的精度.

1 状态方程含未知输入的情况

1.1 基本方程

当状态方程中含有未知输入(如突风、故障、未知的系统误差等)时, Kalman 滤波中的状态方程和量测方程可分别表示为

$$\mathbf{X}_k = \Phi_{k-1} \mathbf{X}_{k-1} + \mathbf{b}_{k-1} + \mathbf{W}_{k-1} \quad (1)$$

$$\mathbf{Z}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{V}_k \quad (2)$$

式中 $\mathbf{X}_k$ 为状态向量; Φ_{k-1} 为状态转移矩阵; $\mathbf{b}_{k-1}$ 为未知输入; $\mathbf{W}_{k-1}$ 为系统噪声向量; $\mathbf{Z}_k$ 为量测向量; $\mathbf{H}_k$ 为量测矩阵; $\mathbf{V}_k$ 为量测噪声向量.假设系统噪声和量测噪声是互不相关的零均值高斯白噪声,方差阵分别为 $\mathbf{Q}_k$ 和 $\mathbf{R}_k$,即 $\mathbf{W}_k$ 和 $\mathbf{V}_k$ 满足

$$\left. \begin{aligned} E[\mathbf{W}_k] &= 0 \\ \text{Cov}[\mathbf{W}_k, \mathbf{W}_j] &= E[\mathbf{W}_k \mathbf{W}_j^T] = \mathbf{Q}_k \delta_{kj} \\ E[\mathbf{V}_k] &= 0 \\ \text{Cov}[\mathbf{V}_k, \mathbf{V}_j] &= E[\mathbf{V}_k \mathbf{V}_j^T] = \mathbf{R}_k \delta_{kj} \\ \text{Cov}[\mathbf{W}_k, \mathbf{V}_j] &= E[\mathbf{W}_k \mathbf{V}_j^T] = 0 \end{aligned} \right\} \quad (3)$$

式中 Cov 为协方差符号; E 为数学期望符号; δ_{kj} 为 δ 函数;当 $k=j$ 时, $\delta_{kj}=1$;当 $k \neq j$ 时, $\delta_{kj}=0$.

1.2 自校准 Kalman 滤波方法

按下面步骤递推,可对式(1)和式(2)给出的状态方程和量测方程进行自校准 Kalman 滤波,

得到 t_k 时刻的状态向量 $\mathbf{X}_k$ 的估计量 $\hat{\mathbf{X}}_k$.

状态一步预测(当 $k>2$ 时)

$$\hat{\mathbf{X}}_{k/k-1} = (\mathbf{I} + \Phi_{k-1}) \hat{\mathbf{X}}_{k-1} - \Phi_{k-2} \hat{\mathbf{X}}_{k-2} \quad (4)$$

当 $k=1,2$ 时, $\hat{\mathbf{X}}_{1/0} = \Phi_0 \hat{\mathbf{X}}_0$, $\hat{\mathbf{X}}_{2/1} = \Phi_1 \hat{\mathbf{X}}_1$.

一步预测误差方差矩阵(当 $k>2$ 时)

$$\begin{aligned} \mathbf{P}_{k/k-1} &= (\mathbf{I} + \Phi_{k-1}) \mathbf{P}_{k-1} (\mathbf{I} + \Phi_{k-1})^T + \\ &\Phi_{k-2} \mathbf{P}_{k-2} \Phi_{k-2}^T - (\mathbf{I} + \Phi_{k-1}) \mathbf{S}_{k-1} \Phi_{k-2}^T - \\ &\Phi_{k-2} \mathbf{S}_{k-1}^T (\mathbf{I} + \Phi_{k-1})^T - (\mathbf{I} + \Phi_{k-1}) \times \\ &(\mathbf{I} - \mathbf{K}_{k-1} \mathbf{H}_{k-1}) \mathbf{Q}_{k-2} - \mathbf{Q}_{k-2} (\mathbf{I} - \mathbf{K}_{k-1} \mathbf{H}_{k-1})^T \times \\ &(\mathbf{I} + \Phi_{k-1})^T + \mathbf{Q}_{k-1} + \mathbf{Q}_{k-2} \end{aligned} \quad (5)$$

当 $k=1,2$ 时, $\mathbf{P}_{k/k-1} = \Phi_{k-1} \mathbf{P}_{k-1} \Phi_{k-1}^T + \mathbf{Q}_{k-1}$.

有时为了简化计算,也可用 $\mathbf{P}_{k/k-1} = \Phi_{k-1} \mathbf{P}_{k-1} \Phi_{k-1}^T + \mathbf{Q}_{k-1}$ 代替式(5)进行近似递推计算.

状态估计

$$\hat{\mathbf{X}}_k = \hat{\mathbf{X}}_{k/k-1} + \mathbf{K}_k (\mathbf{Z}_k - \mathbf{H}_k \hat{\mathbf{X}}_{k/k-1}) \quad (6)$$

估计误差方差矩阵

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k/k-1} (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)^T + \mathbf{K}_k \mathbf{R}_k \mathbf{K}_k^T \quad (7)$$

滤波增益阵

$$\mathbf{K}_k = \mathbf{P}_{k/k-1} \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{k/k-1} \mathbf{H}_k^T + \mathbf{R}_k)^{-1} \quad (8)$$

式中 $\mathbf{I}$ 为单位矩阵,并且

$$\begin{aligned} \mathbf{S}_{k-1} &= (\mathbf{I} - \mathbf{K}_{k-1} \mathbf{H}_{k-1}) [(\mathbf{I} + \Phi_{k-2}) \mathbf{P}_{k-2} - \\ &\Phi_{k-3} \mathbf{S}_{k-2}^T - \mathbf{Q}_{k-3} (\mathbf{I} - \mathbf{K}_{k-2} \mathbf{H}_{k-2})^T] \end{aligned} \quad (9)$$

$$\mathbf{S}_1 = \mathbf{P}_1 \quad (10)$$

证明 由于工程实际中,滤波间隔较短,相邻两个未知输入 $\mathbf{b}_k$ 和 $\mathbf{b}_{k-1}$ 的大小通常相同或相近,即有

$$\mathbf{b}_k \approx \mathbf{b}_{k-1} \quad (11)$$

故式(1)中 $\mathbf{b}_{k-1}$ 的估计为

$$\hat{\mathbf{b}}_{k-1} = \hat{\mathbf{b}}_{k-2} = \hat{\mathbf{X}}_{k-1} - \Phi_{k-2} \hat{\mathbf{X}}_{k-2} \quad (12)$$

将式(12)代入式(1)进行一步预测,则得

$$\hat{\mathbf{X}}_{k/k-1} = (\mathbf{I} + \Phi_{k-1}) \hat{\mathbf{X}}_{k-1} - \Phi_{k-2} \hat{\mathbf{X}}_{k-2} \quad (13)$$

因此一步预测与真实状态之间的误差为

$$\tilde{\mathbf{X}}_{k/k-1} = \mathbf{X}_k - \hat{\mathbf{X}}_{k/k-1} \quad (14)$$

将式(1)代入式(14),得

$$\begin{aligned} \tilde{\mathbf{X}}_{k/k-1} &= (\mathbf{I} + \Phi_{k-1}) \tilde{\mathbf{X}}_{k-1} - \Phi_{k-2} \tilde{\mathbf{X}}_{k-2} + \\ &\mathbf{W}_{k-1} - \mathbf{W}_{k-2} \end{aligned} \quad (15)$$

设 $\mathbf{X}_k$ 的估计 $\hat{\mathbf{X}}_k$ 由式(6)给出,其误差为

$$\tilde{\mathbf{X}}_k = \mathbf{X}_k - \hat{\mathbf{X}}_k \quad (16)$$

即

$$\begin{aligned} \tilde{\mathbf{X}}_k &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \tilde{\mathbf{X}}_{k/k-1} - \mathbf{K}_k \mathbf{V}_k = \\ &(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) [(\mathbf{I} + \Phi_{k-1}) \tilde{\mathbf{X}}_{k-1} - \\ &\Phi_{k-2} \tilde{\mathbf{X}}_{k-2} + \mathbf{W}_{k-1} - \mathbf{W}_{k-2}] - \mathbf{K}_k \mathbf{V}_k \end{aligned} \quad (17)$$

若 $\hat{\mathbf{X}}_{k-1}$ 和 $\hat{\mathbf{X}}_{k-2}$ 具有无偏性,则有

$$E[\hat{\mathbf{X}}_k] = E[\mathbf{X}_k] \quad (18)$$

即式(6)给出的 $\hat{\mathbf{X}}_k$ 是 $\mathbf{X}_k$ 无偏估计量.

因此,一步预测误差方差矩阵为

$$\begin{aligned} P_{k/k-1} &= E[\tilde{\mathbf{X}}_{k/k-1} \tilde{\mathbf{X}}_{k/k-1}^T] = E\left\{[(\mathbf{I} + \Phi_{k-1})\tilde{\mathbf{X}}_{k-1} - \Phi_{k-2}\tilde{\mathbf{X}}_{k-2} + \mathbf{W}_{k-1} - \mathbf{W}_{k-2}] \times \right. \\ &\left. [(\mathbf{I} + \Phi_{k-1})\tilde{\mathbf{X}}_{k-1} - \Phi_{k-2}\tilde{\mathbf{X}}_{k-2} + \mathbf{W}_{k-1} - \mathbf{W}_{k-2}]^T\right\} = (\mathbf{I} + \Phi_{k-1})P_{k-1}(\mathbf{I} + \Phi_{k-1})^T + \Phi_{k-2}P_{k-2}\Phi_{k-2}^T + \\ &\quad Q_{k-1} + Q_{k-2} - (\mathbf{I} + \Phi_{k-1})E(\tilde{\mathbf{X}}_{k-1}\mathbf{W}_{k-2}^T) - E(\mathbf{W}_{k-2}\tilde{\mathbf{X}}_{k-1}^T)(\mathbf{I} + \Phi_{k-1})^T - \\ &\quad (\mathbf{I} + \Phi_{k-1})E(\tilde{\mathbf{X}}_{k-1}\tilde{\mathbf{X}}_{k-2}^T)\Phi_{k-2}^T - \Phi_{k-2}E(\tilde{\mathbf{X}}_{k-2}\tilde{\mathbf{X}}_{k-1}^T)(\mathbf{I} + \Phi_{k-1})^T + (\mathbf{I} + \Phi_{k-1})E(\tilde{\mathbf{X}}_{k-1}\mathbf{W}_{k-1}^T) + \\ &\quad [(\mathbf{I} + \Phi_{k-1})E(\tilde{\mathbf{X}}_{k-1}\mathbf{W}_{k-1}^T)]^T - \Phi_{k-2}E(\tilde{\mathbf{X}}_{k-2}\mathbf{W}_{k-1}^T) - [\Phi_{k-2}E(\tilde{\mathbf{X}}_{k-2}\mathbf{W}_{k-1}^T)]^T + \\ &\quad \Phi_{k-2}E(\tilde{\mathbf{X}}_{k-2}\mathbf{W}_{k-2}^T) + [\Phi_{k-2}E(\tilde{\mathbf{X}}_{k-2}\mathbf{W}_{k-2}^T)]^T \end{aligned} \quad (19)$$

由于

$$E[\tilde{\mathbf{X}}_k \mathbf{W}_k^T] = E\left\{[(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)((\mathbf{I} + \Phi_{k-1})\tilde{\mathbf{X}}_{k-1} - \Phi_{k-2}\tilde{\mathbf{X}}_{k-2} + \mathbf{W}_{k-1} - \mathbf{W}_{k-2}) - \mathbf{K}_k \mathbf{V}_k] \mathbf{W}_k^T\right\} = 0 \quad (20)$$

$$\begin{aligned} E[\tilde{\mathbf{X}}_k \mathbf{W}_{k-1}^T] &= E\left\{[(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)((\mathbf{I} + \Phi_{k-1})\tilde{\mathbf{X}}_{k-1} - \Phi_{k-2}\tilde{\mathbf{X}}_{k-2} + \mathbf{W}_{k-1} - \mathbf{W}_{k-2}) - \mathbf{K}_k \mathbf{V}_k] \mathbf{W}_{k-1}^T\right\} = \\ &\quad (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)Q_{k-1} \\ S_{k-1} &= E[\tilde{\mathbf{X}}_{k-1} \tilde{\mathbf{X}}_{k-2}^T] = \end{aligned} \quad (21)$$

$$\begin{aligned} E\left\{[(\mathbf{I} - \mathbf{K}_{k-1} \mathbf{H}_{k-1})((\mathbf{I} + \Phi_{k-2})\tilde{\mathbf{X}}_{k-2} - \Phi_{k-3}\tilde{\mathbf{X}}_{k-3} + \mathbf{W}_{k-2} - \mathbf{W}_{k-3}) - \mathbf{K}_{k-1} \mathbf{V}_{k-1}] \tilde{\mathbf{X}}_{k-2}^T\right\} = \\ (\mathbf{I} - \mathbf{K}_{k-1} \mathbf{H}_{k-1})[(\mathbf{I} + \Phi_{k-2})P_{k-2} - \Phi_{k-3}E(\tilde{\mathbf{X}}_{k-3}\tilde{\mathbf{X}}_{k-2}^T) - E(\mathbf{W}_{k-3}\tilde{\mathbf{X}}_{k-2}^T)] = \\ (\mathbf{I} - \mathbf{K}_{k-1} \mathbf{H}_{k-1})[(\mathbf{I} + \Phi_{k-2})P_{k-2} - \Phi_{k-3}S_{k-2}^T - Q_{k-3}(\mathbf{I} - \mathbf{K}_{k-2} \mathbf{H}_{k-2})^T] \end{aligned} \quad (22)$$

所以,式(19)可化简为

$$\begin{aligned} P_{k/k-1} &= (\mathbf{I} + \Phi_{k-1})P_{k-1}(\mathbf{I} + \Phi_{k-1})^T + \Phi_{k-2}P_{k-2}\Phi_{k-2}^T - (\mathbf{I} + \Phi_{k-1})S_{k-1}\Phi_{k-2}^T - \Phi_{k-2}S_{k-1}^T(\mathbf{I} + \Phi_{k-1})^T - \\ &\quad (\mathbf{I} + \Phi_{k-1})(\mathbf{I} - \mathbf{K}_{k-1} \mathbf{H}_{k-1})Q_{k-2} - Q_{k-2}(\mathbf{I} - \mathbf{K}_{k-1} \mathbf{H}_{k-1})^T(\mathbf{I} + \Phi_{k-1})^T + Q_{k-1} + Q_{k-2} \end{aligned} \quad (23)$$

估计误差方差矩阵为

$$\begin{aligned} P_k &= E[\tilde{\mathbf{X}}_k \tilde{\mathbf{X}}_k^T] = E\left\{[(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)\tilde{\mathbf{X}}_{k/k-1} - \mathbf{K}_k \mathbf{V}_k] \times \right. \\ &\left. [(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)\tilde{\mathbf{X}}_{k/k-1} - \mathbf{K}_k \mathbf{V}_k]^T\right\} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)P_{k/k-1}(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)^T + \mathbf{K}_k \mathbf{R}_k \mathbf{K}_k^T \end{aligned} \quad (24)$$

增益阵 $\mathbf{K}_k$ 的选取准则是使估计误差方差矩阵 P_k 达到最小. 由式(24)对 $\mathbf{K}_k$ 求偏导, 得

$$\frac{\partial}{\partial \mathbf{K}_k} \text{tr}\{P_k\} = \text{tr}\{(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)P_{k/k-1}(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)^T + \mathbf{K}_k \mathbf{R}_k \mathbf{K}_k^T\} = 0 \quad (25)$$

由式(25)求得增益阵 $\mathbf{K}_k$, 即得式(8).

证毕.

2 量测方程含未知输入的情况

2.1 基本方程

当量测方程中含有未知输入时, Kalman 滤波中的状态方程和量测方程可分别表示为

$$\mathbf{X}_k = \Phi_{k-1} \mathbf{X}_{k-1} + \mathbf{W}_{k-1} \quad (26)$$

$$\mathbf{Z}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{d}_k + \mathbf{V}_k \quad (27)$$

式中 $\mathbf{X}_k$, Φ_{k-1} , $\mathbf{Z}_k$, $\mathbf{H}_k$, $\mathbf{W}_{k-1}$, $\mathbf{V}_k$ 共 6 个量的含义与式(1)和式(2)相同; $\mathbf{d}_k$ 为未知输入.

2.2 自校准 Kalman 滤波方法

首先, 令

$$\mathbf{Y}_k = \mathbf{X}_k + \mathbf{H}_k^+ \mathbf{d}_k \quad (28)$$

代入式(26)和式(27), 则将其变换为

$$\mathbf{Y}_k = \Phi_{k-1} \mathbf{Y}_{k-1} + \mathbf{b}_{k-1} + \mathbf{W}_{k-1} \quad (29)$$

$$\mathbf{Z}_k = \mathbf{H}_k \mathbf{Y}_k + \mathbf{V}_k \quad (30)$$

式中 $\mathbf{H}_k^+$ 为 $\mathbf{H}_k$ 的广义逆, 即 $\mathbf{H}_k \mathbf{H}_k^+ \mathbf{H}_k = \mathbf{H}_k$, 当 $\mathbf{H}_k$ 为低阵时, 即 $\text{rank}(\mathbf{H}_{km \times n}) = m (m \leq n)$ 有 $\mathbf{H}_k \mathbf{H}_k^+ = \mathbf{I}$ (本文中广义逆均按此定义)。并且

$$\mathbf{b}_{k-1} = \mathbf{H}_k^+ \mathbf{d}_k - \Phi_{k-1} \mathbf{H}_{k-1}^+ \mathbf{d}_{k-1} \quad (31)$$

然后, 采用本文第 1 节方法 (同理有 $\mathbf{d}_k \approx \mathbf{d}_{k-1}$), 可对式(29)和式(30)给出的状态方程和量测方程进行自校准 Kalman 滤波, 得到 t_k 时刻向量 $\mathbf{Y}_k$ 的估计量 $\hat{\mathbf{Y}}_k$. 再由下式求得 $t_k (k > 2)$ 时刻状态向量 $\mathbf{X}_k$ 的估计量 $\hat{\mathbf{X}}_k$

$$\hat{\mathbf{X}}_k = \hat{\mathbf{Y}}_k - \mathbf{H}_k^+ \hat{\mathbf{d}}_k \quad (32)$$

$$\hat{\mathbf{d}}_k = r_k \hat{\mathbf{d}}_k^* + (1 - r_k) \hat{\mathbf{d}}_{k-1} \quad (33)$$

$$\hat{d}_k^* = (\mathbf{H}_k^+ - \Phi_{k-1} \mathbf{H}_{k-1}^+)^+ (\hat{\mathbf{Y}}_k - \Phi_{k-1} \hat{\mathbf{Y}}_{k-1}) \quad (34)$$

式中 r_k 为权重系数 ($0 \leq r_k \leq 1$), 且 $r_3 = 1$.

3 状态方程和量测方程均含未知输入的情况

3.1 基本方程

当状态方程和量测方程中均含有未知输入时, Kalman 滤波中的状态方程和量测方程可分别表示为

$$\mathbf{X}_k = \Phi_{k-1} \mathbf{X}_{k-1} + \mathbf{E}_{k-1} \mathbf{d}_{k-1} + \mathbf{W}_{k-1} \quad (35)$$

$$\mathbf{Z}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{G}_k \mathbf{d}_k + \mathbf{V}_k \quad (36)$$

式中 $\mathbf{X}_k, \Phi_{k-1}, \mathbf{Z}_k, \mathbf{H}_k, \mathbf{W}_{k-1}, \mathbf{V}_k$ 共 6 个量的含义与式(1)和式(2)相同; $\mathbf{d}_k$ 为未知输入; $\mathbf{E}_{k-1}$ 和 $\mathbf{G}_k$ 为已知的输入矩阵.

3.2 自校准 Kalman 滤波方法

同样, 令

$$\mathbf{Y}_k = \mathbf{X}_k + \mathbf{H}_k^+ \mathbf{G}_k \mathbf{d}_k \quad (37)$$

代入式(35)和式(36), 则将其变换为

$$\mathbf{Y}_k = \Phi_{k-1} \mathbf{Y}_{k-1} + \mathbf{b}_{k-1} + \mathbf{W}_{k-1} \quad (38)$$

$$\mathbf{Z}_k = \mathbf{H}_k \mathbf{Y}_k + \mathbf{V}_k \quad (39)$$

式中

$$\mathbf{b}_{k-1} = \mathbf{H}_k^+ \mathbf{G}_k \mathbf{d}_k - \Phi_{k-1} \mathbf{H}_{k-1}^+ \mathbf{G}_{k-1} \mathbf{d}_{k-1} + \mathbf{E}_{k-1} \mathbf{d}_{k-1} \quad (40)$$

采用本文第 1 节方法, 可对式(38)和式(39)给出的状态方程和量测方程进行自校准 Kalman 滤波, 得到 t_k 时刻向量 $\mathbf{Y}_k$ 的估计量 $\hat{\mathbf{Y}}_k$. 再由下式求得 t_k ($k > 2$) 时刻状态向量 $\mathbf{X}_k$ 的估计量 $\hat{\mathbf{X}}_k$

$$\hat{\mathbf{X}}_k = \hat{\mathbf{Y}}_k - \mathbf{H}_k^+ \mathbf{G}_k \hat{\mathbf{d}}_k \quad (41)$$

$$\hat{\mathbf{d}}_k = r_k \hat{\mathbf{d}}_k^* + (1 - r_k) \hat{\mathbf{d}}_{k-1} \quad (42)$$

$$\hat{\mathbf{d}}_k^* = (\mathbf{H}_k^+ \mathbf{G}_k - \Phi_{k-1} \mathbf{H}_{k-1}^+ \mathbf{G}_{k-1} + \mathbf{E}_{k-1})^+ \times (\hat{\mathbf{Y}}_k - \Phi_{k-1} \hat{\mathbf{Y}}_{k-1}) \quad (43)$$

4 计算机仿真验证

文献[20]给出了某飞行器线性化动态模型, 状态变量为侧滑角、倾斜角、滚动角速度和偏航角速度, 控制输入为方向舵偏航角和副翼转向角, 未知输入干扰为突风.

$$\mathbf{X}_k = \Phi_{k-1} \mathbf{X}_{k-1} + \mathbf{\Gamma}_{k-1} \mathbf{u}_{k-1} + \mathbf{E}_{k-1} \mathbf{d}_{k-1} + \mathbf{W}_{k-1} \quad (44)$$

$$\mathbf{Z}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{G}_k \mathbf{d}_k + \mathbf{V}_k \quad (45)$$

设有关参数如下:

$$\Phi_{k-1} =$$

$$\begin{bmatrix} -0.2100 & 0.0340 & -0.0011 & -0.9900 \\ 0 & 0 & 1.0 & 0 \\ -5.555 & 0 & -1.8900 & 0.3900 \\ 2.4300 & 0 & -0.0340 & -2.9800 \end{bmatrix}$$

$$\mathbf{\Gamma}_{k-1} = \begin{bmatrix} 0.0300 & 0 \\ 0 & 0 \\ 0.3600 & -1.6000 \\ -0.9500 & -0.0320 \end{bmatrix}$$

$$\mathbf{E}_{k-1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{G}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.2 \end{bmatrix} \quad \mathbf{u}_{k-1} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$\mathbf{H}_k = \mathbf{I}, \mathbf{Q}_k = 0.001^2 \mathbf{I}, \mathbf{R}_k = 0.001^2 \mathbf{I}$. 已知输入控制量 u_1 和 u_2 由图 1 和图 2 给出, 未知输入 $\mathbf{d}_k$ 由图 3 给出.

分别采用传统 Kalman 滤波方法和本文自校准 Kalman 滤波方法进行滤波, 状态初始值取为

$\mathbf{X}_0 = [0.1 \ 0 \ 0 \ 0]^T, \hat{\mathbf{X}}_0 = [0 \ 0 \ 0 \ 0]^T$; 初始估计均方误差阵为 $\mathbf{P}_0 = 100 \mathbf{I}$; 权重系数 $r_k = 0.5$. 图 4~图 7 给出了两种滤波结果与真值的误差, 从中可以看到, 自校准 Kalman 滤波方法得到的状态估计与真值吻合较好, 能够降低未知输入对飞行器状态估计的影响, 提高滤波的精度, 很好地跟踪状态向量 $\mathbf{X}_k$ 的变化. 而 Kalman 滤波方法受未知输入的影响, 与真值的误差已不在零附近波动, 误差较大. 表 1 进一步给出了两种滤波方法状态估计误差的均值和方差, 从中可知, 自校准 Kalman 滤波方法误差的均值和方差分别比 Kalman 滤波方法降低了 400% 和 300% 以上.

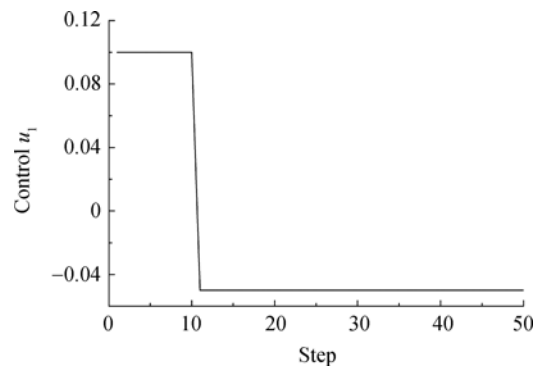
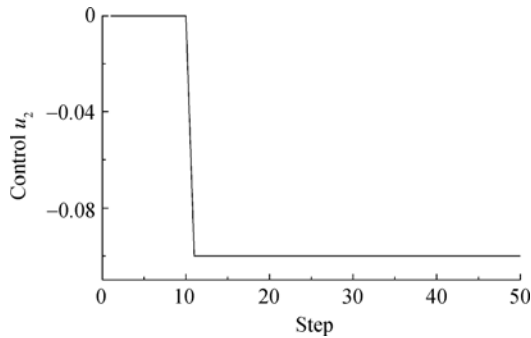
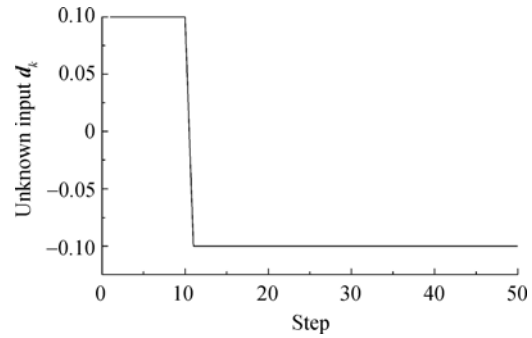
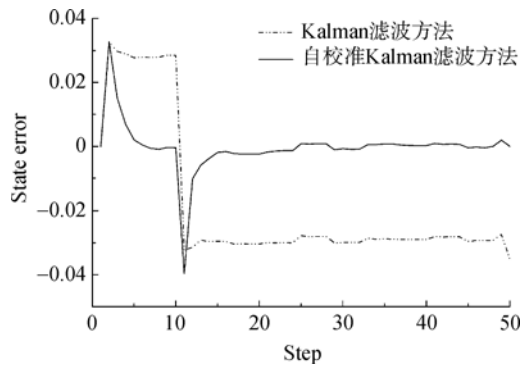
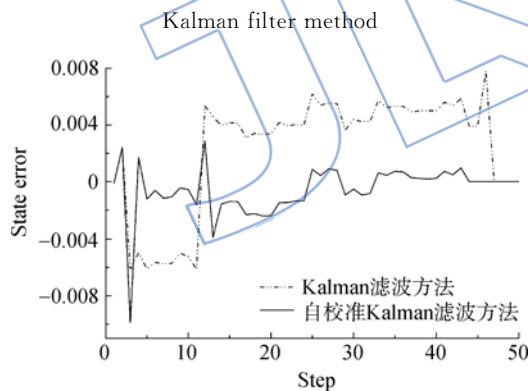
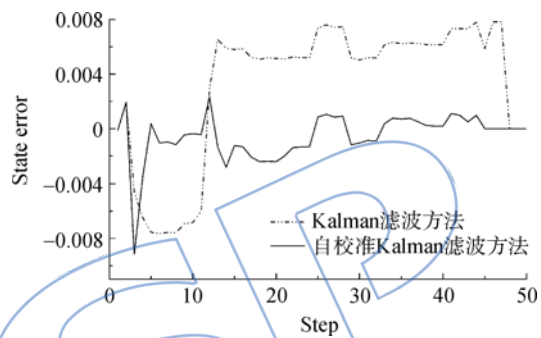


图 1 控制输入量 u_1

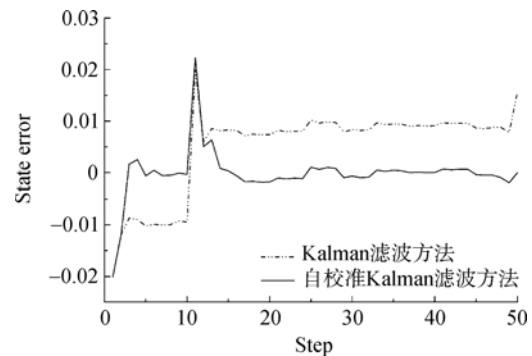
Fig.1 Control input u_1

图 2 控制输入量 u_k Fig. 2 Control input u_k 图 3 未知输入量 d_k Fig. 3 Unknown input d_k 图 4 Kalman 滤波方法和自校准 Kalman 滤波方法估计的状态 x_1 误差比较Fig. 4 Comparison of estimated errors of state x_1 by Kalman filter method and self-calibration图 6 Kalman 滤波方法和自校准 Kalman 滤波方法估计的状态 x_3 误差比较Fig. 6 Comparison of estimated errors of state x_3 by Kalman filter method and self-calibration

Kalman filter method

图 5 Kalman 滤波方法和自校准 Kalman 滤波方法估计的状态 x_2 误差比较Fig. 5 Comparison of estimated errors of state x_2 by Kalman filter method and self-calibration

Kalman filter method

图 7 Kalman 滤波方法和自校准 Kalman 滤波方法估计的状态 x_4 误差比较Fig. 7 Comparison of estimated errors of state x_4 by Kalman filter method and self-calibration

Kalman filter method

表 1 Kalman 滤波方法和自校准 Kalman 滤波方法估计误差的均值与方差

Table 1 Mean and variance of estimated state errors by Kalman filter method and self-calibration Kalman filter method

状态	Kalman 滤波方法		自校准 Kalman 滤波方法		状态	Kalman 滤波方法		自校准 Kalman 滤波方法	
	均值	方差	均值	方差		均值	方差	均值	方差
1	-0.0184	0.0005	-0.0003	0.00006	3	0.0023	0.00002	-0.0005	0.000003
2	0.0032	0.00006	-0.0006	0.000003	4	0.0051	0.00007	-0.0001	0.00002

5 结 论

1) 在导航、信号处理和故障诊断等领域,系统常常受到未知输入的干扰.传统 Kalman 滤波无法消除这些未知输入的影响,导致较大的误差,本文提出的自校准 Kalman 滤波则能够消除或补偿这些未知输入的影响,从而提高滤波精度.

2) 由于实际问题的复杂性以及系统内部故障和外部干扰,状态方程要精确描述实际情况是非常困难的,也就是说状态方程中常常含有各种各样的未知输入.文中第1节给出的自校准 Kalman 滤波方法,能够成功地消除状态方程中未知输入的影响,提高滤波精度.

3) 由于环境因素的影响、测量设备的不稳定性、模型和参数的选取不当等原因,常常造成测量数据含有未知系统误差,文中第2节给出的自校准 Kalman 滤波方法能够对量测方程中这种未知的系统误差进行估计和补偿.

4) 在火星等行星探测中,突风、升阻比、大气密度、弹道系数等许多因素往往对状态方程和量测方程均产生影响,当某些因素的参数值与实际不一致或突然变化时,常常带来未知输入.本文第3节给出的自校准 Kalman 滤波方法能够对这种未知输入进行估计和补偿,提高滤波精度.

5) 本文方法得到的状态估计量具有无偏性,并且计算简单,便于工程应用.

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