

非线性状态方程自校准滤波方法

傅惠民, 杨海峰, 肖梦丽, 肖 强
(北京航空航天大学 小样本技术研究中心, 北京 100191)

摘 要: 针对工程实际中遇到的非线性系统状态方程中含未知输入(如环境因素的影响、模型和参数选取不当等)的情况,采用自校准技术,基于秩滤波与无迹 Kalman 滤波算法提出了一种非线性状态方程自校准滤波方法,并分别讨论了自校准秩滤波(SRF)与自校准无迹 Kalman 滤波(SUKF)两种情况。大量仿真结果和工程应用表明:与无迹 Kalman 滤波(UKF)相比,该方法通过对系统状态方程中的未知输入进行自动估计和补偿,改善了系统受未知输入影响下的滤波效果,从算例中可以看到,估计精度至少提高了 80%,且计算简单,便于工程应用。

关 键 词: 自校准滤波; 非线性滤波; 秩滤波; 无迹 Kalman 滤波; 故障诊断; 未知输入

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Nonlinear state equation self-calibration filtering method

FU Huimin, YANG Haifeng, XIAO Mengli, XIAO Qiang

(Research Center of Small Sample Technology,
Beijing University of Aeronautics and Astronautics, Beijing 100191, China)

Abstract: In view of the situation that system state equations are influenced by unknown inputs (such as environmental influence, improper selection of models or parameters, and etc.), a nonlinear self-calibration filtering recursive method for state equations with unknown inputs was proposed based on two nonlinear Kalman filtering methods, the self-calibration rank filter (SRF) and the self-calibration unscented Kalman filter (SUKF) were discussed respectively. According to numerous numerical simulation results and engineering applications, by estimating and compensating the unknown inputs in state equations automatically, the proposed algorithm can improve the filtering effect of the system under the influence of unknown inputs, and the estimation accuracy increased by 80% when compared with the unscented Kalman filtering method (UKF). Moreover, the calculation was simple and convenient for engineering applications.

Key words: self-calibration filter; nonlinear filter; rank filter;
unscented Kalman filter; fault diagnosis; unknown input

自 1960 年 Kalman 提出 Kalman 滤波方法^[1]以来,该方法在故障诊断、信号处理、导航等工程领域内得到了广泛、深入的应用。基于标准 Kalman 滤波方法,研究人员先后发展了扩展 Kal-

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作者简介:傅惠民(1956—),男,教授、博士生导师,博士,主要从事小样本信息技术、软校准技术、数据融合方法、可靠性及滤波理论方面的研究。

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man 滤波(extended Kalman filter, EKF)^[2-5]、无迹 Kalman 滤波(unscented Kalman filter, UKF)^[6-9]、粒子滤波(particle filter, PF)^[10-12]等方法用以解决非线性系统的滤波问题。然而, EKF 只能处理弱非线性的系统, UKF 只能处理服从高斯分布的系统, PF 虽然能够处理非高斯、非线性系统, 却存在粒子退化与粒子贫化问题。傅惠民等基于秩采样, 推导、建立了适用于非高斯、非线性系统的秩滤波方法(rank filter, RF)^[13-16], 该方法计算简便, 易于工程应用。

传统的滤波方法是针对精确的系统方程建立的。但是, 在工程实际中, 由于环境因素的影响、模型和参数选取不当等原因, 状态方程往往存在系统误差, 这种误差未知且较平稳(未知输入)^[17-19], 从而导致滤波精度下降。因此, 如何补偿状态方程未知输入的影响是目前国内外研究的难点与热点。针对此问题, 傅惠民等提出了自校准 Kalman 滤波方法(self-calibration Kalman filter, SKF)^[20], 使得未知输入的影响自动得到补偿。

本文将自校准技术引入到非线性系统的滤波问题中来, 将其分别与 RF 和 UKF 相结合, 推导得到了自校准秩滤波(self-calibration rank filter, SRF)方法与自校准无迹 Kalman 滤波方法(self-calibration unscented Kalman filter, SUKF), 统称作非线性状态方程自校准滤波(nonlinear self-calibration Kalman filter)方法。与传统的滤波方法相比, 本文方法自动补偿了未知输入的影响, 且适用于强非线性系统, 形式简单, 便于工程实际应用。仿真结果表明, 本文方法在应对未知输入影响问题时, 滤波精度更高, 稳定性与抗干扰能力更强。

1 自校准秩滤波方法

1.1 加性噪声情况

1.1.1 基本方程

工程中含有未知输入的非线性离散系统一般可表示为

$$\mathbf{X}_k = \mathbf{f}(\mathbf{X}_{k-1}) + \mathbf{b}_{k-1} + \mathbf{W}_{k-1} \quad (1)$$

$$\mathbf{Z}_k = \mathbf{h}(\mathbf{X}_k) + \mathbf{V}_k \quad (2)$$

式中 $\mathbf{X}_k$ 表示系统的状态向量, $\mathbf{Z}_k$ 表示系统量测向量, $\mathbf{f}(\cdot)$ 和 $\mathbf{h}(\cdot)$ 为非线性向量函数, $\mathbf{b}_k$ 表示未知输入, $\mathbf{W}_k$ 与 $\mathbf{V}_k$ 分别为系统噪声向量和量测噪声向量, 其方差矩阵分别为 $\mathbf{Q}_k$ 和 $\mathbf{R}_k$, 并且满足

$$\begin{cases} E[\mathbf{W}_k] = \mathbf{0} \\ \text{Cov}[\mathbf{W}_k, \mathbf{W}_j] = E[\mathbf{W}_k \mathbf{W}_j^T] = \mathbf{Q}_k \delta_{kj} \\ E[\mathbf{V}_k] = \mathbf{0} \\ \text{Cov}[\mathbf{V}_k, \mathbf{V}_j] = E[\mathbf{V}_k \mathbf{V}_j^T] = \mathbf{R}_k \delta_{kj} \\ \text{Cov}[\mathbf{W}_k, \mathbf{V}_j] = E[\mathbf{W}_k \mathbf{V}_j^T] = \mathbf{0} \end{cases} \quad (3)$$

式中 $\text{Cov}[\cdot]$ 为协方差, $E[\cdot]$ 为数学期望, δ_{kj} 为 δ 函数, 当 $k=j$ 时, $\delta_{kj}=1$; 当 $k \neq j$ 时, $\delta_{kj}=0$ 。

1.1.2 自校准秩滤波递推步骤

由于工程中相邻两步滤波之间的时间间隔很短, 所以它们的未知输入也往往相等或近似相等, 即有 $\mathbf{b}_k \approx \mathbf{b}_{k-1}$ 。在此基础上, 结合 RF 递推过程, 可以证明, SRF 方法由第 $k-1$ 步状态估计推出第 k 步状态估计的步骤如下:

滤波初始化条件为

$$\hat{\mathbf{X}}_0 = E[\mathbf{X}_0] \quad (4)$$

$$\mathbf{P}_0 = E[(\mathbf{X}_0 - \hat{\mathbf{X}}_0)(\mathbf{X}_0 - \hat{\mathbf{X}}_0)^T] \quad (5)$$

1) 时间更新

秩采样点集 $\{\chi_i\}$, 当 $k=1, 2$ 时

$$\begin{aligned} \chi_{k-1,i} = & \begin{cases} \hat{\mathbf{X}}_{k-1} + u_{p_1}(\sqrt{\mathbf{P}_{k-1}})_i & i = 1, 2, \dots, n \\ \hat{\mathbf{X}}_{k-1} - u_{p_1}(\sqrt{\mathbf{P}_{k-1}})_{i-n} & i = n+1, \dots, 2n \\ \hat{\mathbf{X}}_{k-1} + u_{p_2}(\sqrt{\mathbf{P}_{k-1}})_{i-2n} & i = 2n+1, \dots, 3n \\ \hat{\mathbf{X}}_{k-1} - u_{p_2}(\sqrt{\mathbf{P}_{k-1}})_{i-3n} & i = 3n+1, \dots, 4n \end{cases} \quad (6) \end{aligned}$$

$$\mathbf{X}_{k/(k-1),i} = \mathbf{f}(\chi_{k-1,i}) \quad (7)$$

当 $k \geq 3$ 时

$$\begin{aligned} \chi_{k-2,i} = & \begin{cases} \hat{\mathbf{X}}_{k-2} + u_{p_1}(\sqrt{\mathbf{P}_{k-2}})_i & i = 1, 2, \dots, n \\ \hat{\mathbf{X}}_{k-2} - u_{p_1}(\sqrt{\mathbf{P}_{k-2}})_{i-n} & i = n+1, \dots, 2n \\ \hat{\mathbf{X}}_{k-2} + u_{p_2}(\sqrt{\mathbf{P}_{k-2}})_{i-2n} & i = 2n+1, \dots, 3n \\ \hat{\mathbf{X}}_{k-2} - u_{p_2}(\sqrt{\mathbf{P}_{k-2}})_{i-3n} & i = 3n+1, \dots, 4n \end{cases} \quad (8) \end{aligned}$$

$$\begin{aligned} \chi_{k-1,i} = & \begin{cases} \hat{\mathbf{X}}_{k-1} + u_{p_1}(\sqrt{\mathbf{P}_{k-1}})_i & i = 1, 2, \dots, n \\ \hat{\mathbf{X}}_{k-1} - u_{p_1}(\sqrt{\mathbf{P}_{k-1}})_{i-n} & i = n+1, \dots, 2n \\ \hat{\mathbf{X}}_{k-1} + u_{p_2}(\sqrt{\mathbf{P}_{k-1}})_{i-2n} & i = 2n+1, \dots, 3n \\ \hat{\mathbf{X}}_{k-1} - u_{p_2}(\sqrt{\mathbf{P}_{k-1}})_{i-3n} & i = 3n+1, \dots, 4n \end{cases} \quad (9) \end{aligned}$$

$$\mathbf{X}_{k/(k-1),i} = \mathbf{f}(\chi_{k-1,i}) + \chi_{k-1,i} - \mathbf{f}(\chi_{k-2,i}) \quad (10)$$

式中 n 为状态向量 $\mathbf{X}_k$ 的维度,标准正态偏量

$u_{p_1}=0.4823, u_{p_2}=1.1281, \left(\sqrt{\mathbf{P}_{k-1}}\right)_i$ 表示 $\mathbf{P}_{k-1}$

平方根的第 i 列向量^[9]。

状态一步预测值

$$\hat{\mathbf{X}}_{k/(k-1)} = \frac{1}{4n} \sum_{i=1}^{4n} \mathbf{X}_{k/(k-1),i} \quad (11)$$

一步预测误差方差矩阵

$$\mathbf{P}_{k/(k-1)} = \frac{1}{\omega} \sum_{i=1}^{4n} \left[(\mathbf{X}_{k/(k-1),i} - \hat{\mathbf{X}}_{k/(k-1)}) \times (\mathbf{X}_{k/(k-1),i} - \hat{\mathbf{X}}_{k/(k-1)})^T \right] + \mathbf{Q}_{k-1} \quad (12)$$

式中 $\omega = 2(u_{p_1}^2 + u_{p_2}^2) = 3.0105$ 为协方差权重系数。

2) 量测更新

重新秩采样点集 $\{\chi_{k/(k-1),i}\}$

$$\chi_{k/(k-1),i} = \begin{cases} \hat{\mathbf{X}}_{k/(k-1)} + u_{p_1} \left(\sqrt{\mathbf{P}_{k/(k-1)}} \right)_i & i = 1, 2, \dots, n \\ \hat{\mathbf{X}}_{k/(k-1)} - u_{p_1} \left(\sqrt{\mathbf{P}_{k/(k-1)}} \right)_{i-n} & i = n+1, \dots, 2n \\ \hat{\mathbf{X}}_{k/(k-1)} + u_{p_2} \left(\sqrt{\mathbf{P}_{k/(k-1)}} \right)_{i-2n} & i = 2n+1, \dots, 3n \\ \hat{\mathbf{X}}_{k/(k-1)} - u_{p_2} \left(\sqrt{\mathbf{P}_{k/(k-1)}} \right)_{i-3n} & i = 3n+1, \dots, 4n \end{cases} \quad (13)$$

状态估计值

$$\hat{\mathbf{X}}_k = \hat{\mathbf{X}}_{k/(k-1)} + \mathbf{K}_k (\mathbf{Z}_k - \hat{\mathbf{Z}}_{k/(k-1)}) \quad (14)$$

状态估计误差方差矩阵

$$\mathbf{P}_k = \mathbf{P}_{k/(k-1)} - \mathbf{K}_k \mathbf{P}_{ZZ} \mathbf{K}_k^T \quad (15)$$

式中

$$\mathbf{Z}_{k/(k-1),i} = \mathbf{h}(\chi_{k/(k-1),i}) \quad (16)$$

$$\hat{\mathbf{Z}}_{k/(k-1)} = \frac{1}{4n} \sum_{i=1}^{4n} \mathbf{Z}_{k/(k-1),i} \quad (17)$$

$$\mathbf{P}_{ZZ} = \frac{1}{\omega} \sum_{i=1}^{4n} \left[(\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)}) \times (\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)})^T \right] + \mathbf{R}_k \quad (18)$$

$$\mathbf{P}_{XZ} = \frac{1}{\omega} \sum_{i=1}^{4n} \left[(\chi_{k/(k-1),i} - \hat{\mathbf{X}}_{k/(k-1)}) \times (\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)})^T \right] \quad (19)$$

$$\mathbf{K}_k = \mathbf{P}_{XZ} \mathbf{P}_{ZZ}^{-1} \quad (20)$$

1.2 非加性噪声情况

1.2.1 基本方程

当系统噪声与量测噪声为非加性噪声时,状态方程含未知输入的非线性离散系统一般为

$$\mathbf{X}_k = \mathbf{f}(\mathbf{X}_{k-1}, \mathbf{W}_{k-1}) + \mathbf{b}_{k-1} \quad (21)$$

$$\mathbf{Z}_k = \mathbf{h}(\mathbf{X}_k, \mathbf{V}_k) \quad (22)$$

式中各参量的意义与式(1)和式(2)中相同。

1.2.2 非加性噪声自校准秩滤波递推步骤

对于式(21)和式(22)中的非加性噪声,采用状态扩维的形式进行处理,扩维后的状态向量为

$$\mathbf{X}_{k-1}^a = [\mathbf{X}_{k-1}^T, \mathbf{W}_{k-1}^T, \mathbf{V}_k^T]^T \quad (23)$$

设系统的状态初始条件为

$$\hat{\mathbf{X}}_0 = E[\mathbf{X}_0] \quad (24)$$

$$\mathbf{P}_0 = E[(\mathbf{X}_0 - \hat{\mathbf{X}}_0)(\mathbf{X}_0 - \hat{\mathbf{X}}_0)^T] \quad (25)$$

则相应可得扩维后系统的状态初始条件为

$$\hat{\mathbf{X}}_0^a = E[\mathbf{X}_0^a] = [\hat{\mathbf{X}}_0^T, \mathbf{0}, \mathbf{0}]^T \quad (26)$$

$$\mathbf{P}_0^a = E[(\mathbf{X}_0^a - \hat{\mathbf{X}}_0^a)(\mathbf{X}_0^a - \hat{\mathbf{X}}_0^a)^T] = \begin{bmatrix} \mathbf{P}_0 & & \\ & \mathbf{Q}_0 & \\ & & \mathbf{R}_1 \end{bmatrix} \quad (27)$$

1) 时间更新

秩采样点集 $\{\chi_i^a\}$, 当 $k=1, 2$ 时

$$\chi_{k-1,i}^a = \begin{cases} \hat{\mathbf{X}}_{k-1}^a + u_{p_1} \left(\sqrt{\mathbf{P}_{k-1}^a} \right)_i & i = 1, 2, \dots, n \\ \hat{\mathbf{X}}_{k-1}^a - u_{p_1} \left(\sqrt{\mathbf{P}_{k-1}^a} \right)_{i-n} & i = n+1, \dots, 2n \\ \hat{\mathbf{X}}_{k-1}^a + u_{p_2} \left(\sqrt{\mathbf{P}_{k-1}^a} \right)_{i-2n} & i = 2n+1, \dots, 3n \\ \hat{\mathbf{X}}_{k-1}^a - u_{p_2} \left(\sqrt{\mathbf{P}_{k-1}^a} \right)_{i-3n} & i = 3n+1, \dots, 4n \end{cases} \quad (28)$$

$$\mathbf{X}_{k/(k-1),i} = \mathbf{f}(\chi_{k-1,i}^a, \chi_{k-1,i}^w) \quad (29)$$

当 $k \geq 3$ 时

$$\chi_{k-2,i}^a = \begin{cases} \hat{\mathbf{X}}_{k-2}^a + u_{p_1} \left(\sqrt{\mathbf{P}_{k-2}^a} \right)_i & i = 1, 2, \dots, n \\ \hat{\mathbf{X}}_{k-2}^a - u_{p_1} \left(\sqrt{\mathbf{P}_{k-2}^a} \right)_{i-n} & i = n+1, \dots, 2n \\ \hat{\mathbf{X}}_{k-2}^a + u_{p_2} \left(\sqrt{\mathbf{P}_{k-2}^a} \right)_{i-2n} & i = 2n+1, \dots, 3n \\ \hat{\mathbf{X}}_{k-2}^a - u_{p_2} \left(\sqrt{\mathbf{P}_{k-2}^a} \right)_{i-3n} & i = 3n+1, \dots, 4n \end{cases} \quad (30)$$

$$\chi_{k-1,i}^a = \begin{cases} \hat{\mathbf{X}}_{k-1}^a + u_{p_1} \left(\sqrt{\mathbf{P}_{k-1}^a} \right)_i & i = 1, 2, \dots, n \\ \hat{\mathbf{X}}_{k-1}^a - u_{p_1} \left(\sqrt{\mathbf{P}_{k-1}^a} \right)_{i-n} & i = n+1, \dots, 2n \\ \hat{\mathbf{X}}_{k-1}^a + u_{p_2} \left(\sqrt{\mathbf{P}_{k-1}^a} \right)_{i-2n} & i = 2n+1, \dots, 3n \\ \hat{\mathbf{X}}_{k-1}^a - u_{p_2} \left(\sqrt{\mathbf{P}_{k-1}^a} \right)_{i-3n} & i = 3n+1, \dots, 4n \end{cases} \quad (31)$$

$$\mathbf{X}_{k/(k-1),i} = f(\boldsymbol{\chi}_{k-1,i}^x, \boldsymbol{\chi}_{k-1,i}^w) + \boldsymbol{\chi}_{k-1,i}^x - f(\boldsymbol{\chi}_{k-2,i}^x, \boldsymbol{\chi}_{k-2,i}^w) \quad (32)$$

式中 n 为扩维后状态向量 $\mathbf{X}_k^n$ 的维度, $\boldsymbol{\chi}_{k-1,i}^x$ 、 $\boldsymbol{\chi}_{k-1,i}^w$ 、 $\boldsymbol{\chi}_{k-1,i}^v$ 分别为与状态向量 $\mathbf{X}_{k-1}$ 、系统噪声向量 $\mathbf{W}_{k-1}$ 、量测噪声向量 $\mathbf{V}_k$ 相对应的采样点。

状态一步预测值

$$\hat{\mathbf{X}}_{k/(k-1)} = \frac{1}{4n} \sum_{i=1}^{4n} \mathbf{X}_{k/(k-1),i} \quad (33)$$

一步预测误差方差矩阵

$$\mathbf{P}_{k/(k-1)} = \frac{1}{\omega} \sum_{i=1}^{4n} \left[(\mathbf{X}_{k/(k-1),i} - \hat{\mathbf{X}}_{k/(k-1)}) \times (\mathbf{X}_{k/(k-1),i} - \hat{\mathbf{X}}_{k/(k-1)})^T \right] \quad (34)$$

2) 量测更新

状态估计值

$$\hat{\mathbf{X}}_k = \hat{\mathbf{X}}_{k/(k-1)} + \mathbf{K}_k (\mathbf{Z}_k - \hat{\mathbf{Z}}_{k/(k-1)}) \quad (35)$$

状态估计误差方差矩阵

$$\mathbf{P}_k = \mathbf{P}_{k/(k-1)} - \mathbf{K}_k \mathbf{P}_{ZZ} \mathbf{K}_k^T \quad (36)$$

式中

$$\mathbf{Z}_{k/(k-1),i} = \mathbf{h}(\mathbf{X}_{k/(k-1),i}, \boldsymbol{\chi}_{k,i}^v) \quad (37)$$

$$\hat{\mathbf{Z}}_{k/(k-1)} = \frac{1}{4n} \sum_{i=1}^{4n} \mathbf{Z}_{k/(k-1),i} \quad (38)$$

$$\mathbf{P}_{ZZ} = \frac{1}{\omega} \sum_{i=1}^{4n} \left[(\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)}) \times (\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)})^T \right] \quad (39)$$

$$\mathbf{P}_{XZ} = \frac{1}{\omega} \sum_{i=1}^{4n} \left[(\mathbf{X}_{k/(k-1),i} - \hat{\mathbf{X}}_{k/(k-1)}) \times (\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)})^T \right] \quad (40)$$

$$\mathbf{K}_k = \mathbf{P}_{XZ} \mathbf{P}_{ZZ}^{-1} \quad (41)$$

2 自校准无迹 Kalman 滤波方法

2.1 加性噪声情况

在假设 $\mathbf{b}_k \approx \mathbf{b}_{k-1}$ 的基础上, 结合 UKF 递推过程, 可以证明, 对系统式(1)和式(2)进行 SUKF 滤波, 由第 $k-1$ 步状态估计推出第 k 步状态估计的步骤如下:

滤波初始化条件由式(4)和式(5)计算。

1) 时间更新

Sigma 点集 $\{\boldsymbol{\chi}_i\}$, 当 $k=1, 2$ 时

$$\boldsymbol{\chi}_{k-1,i} =$$

$$\begin{cases} \hat{\mathbf{X}}_{k-1} - [\sqrt{(n+\lambda)\mathbf{P}_{k-1}}]_i & i = 1, 2, \dots, n \\ \hat{\mathbf{X}}_{k-1} & i = n+1 \\ \hat{\mathbf{X}}_{k-1} + [\sqrt{(n+\lambda)\mathbf{P}_{k-1}}]_i & i = n+2, \dots, 2n+1 \end{cases} \quad (42)$$

$$\mathbf{X}_{k/(k-1)}^i = f(\boldsymbol{\chi}_{k-1,i}) \quad (43)$$

当 $k \geq 3$ 时

$$\begin{cases} \boldsymbol{\chi}_{k-2,i} = \hat{\mathbf{X}}_{k-2} - [\sqrt{(n+\lambda)\mathbf{P}_{k-2}}]_i & i = 1, 2, \dots, n \\ \hat{\mathbf{X}}_{k-2} & i = n+1 \\ \hat{\mathbf{X}}_{k-2} + [\sqrt{(n+\lambda)\mathbf{P}_{k-2}}]_i & i = n+2, \dots, 2n+1 \end{cases} \quad (44)$$

$$\begin{cases} \boldsymbol{\chi}_{k-1,i} = \hat{\mathbf{X}}_{k-1} - [\sqrt{(n+\lambda)\mathbf{P}_{k-1}}]_i & i = 1, 2, \dots, n \\ \hat{\mathbf{X}}_{k-1} & i = n+1 \\ \hat{\mathbf{X}}_{k-1} + [\sqrt{(n+\lambda)\mathbf{P}_{k-1}}]_i & i = n+2, \dots, 2n+1 \end{cases} \quad (45)$$

$$\mathbf{X}_{k/(k-1),i} = f(\boldsymbol{\chi}_{k-1,i}) + \boldsymbol{\chi}_{k-1,i} - f(\boldsymbol{\chi}_{k-2,i}) \quad (46)$$

式中 $\lambda = \alpha^2(n+\kappa) - n$ 为 Sigma 点的比例参数, n 为状态向量 $\mathbf{X}_k$ 的维度, κ 为调节参数, 一般取 0 或 $3-n$, α 为另一调节参数。

状态一步预测值

$$\hat{\mathbf{X}}_{k/(k-1)} = \sum_{i=1}^{2n+1} (w_m^i \mathbf{X}_{k/(k-1),i}) \quad (47)$$

式中 w_m^i 为均值权重系数, 其计算公式为

$$w_m^i = \begin{cases} \frac{1}{2(n+\lambda)} & i = 1, 2, \dots, 2n+1 \\ \frac{\lambda}{n+\lambda} & i = n+1 \end{cases} \quad (48)$$

一步预测误差方差矩阵

$$\mathbf{P}_{k/(k-1)} = \sum_{i=1}^{2n+1} \left[w_c^i (\mathbf{X}_{k/(k-1),i} - \hat{\mathbf{X}}_{k/(k-1)}) \times (\mathbf{X}_{k/(k-1),i} - \hat{\mathbf{X}}_{k/(k-1)})^T \right] + \mathbf{Q}_{k-1} \quad (49)$$

式中 w_c^i 为协方差权重系数, 其计算公式为

$$w_c^i = \begin{cases} \frac{1}{2(n+\lambda)} & i = 1, 2, \dots, 2n+1 \\ \frac{\lambda}{n+\lambda} + (1-\alpha^2+\beta) & i = n+1 \end{cases} \quad (50)$$

2) 量测更新

新 Sigma 点集 $\{\boldsymbol{\chi}_{k/(k-1),i}\}$

$$\begin{cases} \boldsymbol{\chi}_{k/(k-1),i} = \hat{\mathbf{X}}_{k/(k-1)} - [\sqrt{(n+\lambda)\mathbf{P}_{k/(k-1)}}]_i & i = 1, 2, \dots, n \\ \hat{\mathbf{X}}_{k/(k-1)} & i = n+1 \\ \hat{\mathbf{X}}_{k/(k-1)} + [\sqrt{(n+\lambda)\mathbf{P}_{k/(k-1)}}]_i & i = n+2, \dots, 2n+1 \end{cases} \quad (51)$$

状态估计值

$$\hat{\mathbf{X}}_k = \hat{\mathbf{X}}_{k/(k-1)} + \mathbf{K}_k (\mathbf{Z}_k - \hat{\mathbf{Z}}_{k/(k-1)}) \quad (52)$$

状态估计误差方差矩阵

$$\mathbf{P}_k = \mathbf{P}_{k/(k-1)} - \mathbf{K}_k \mathbf{P}_{ZZ} \mathbf{K}_k^T \quad (53)$$

式中

$$\mathbf{Z}_{k/(k-1),i} = \mathbf{h}(\mathbf{x}_{k/(k-1),i}) \quad (54)$$

$$\hat{\mathbf{Z}}_{k/(k-1)} = \sum_{i=1}^{2n+1} (\omega_m^i \mathbf{Z}_{k/(k-1),i}) \quad (55)$$

$$\mathbf{P}_{ZZ} = \sum_{i=1}^{2n+1} \left[\omega_c^i (\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)}) \times (\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)})^T \right] + \mathbf{R}_k \quad (56)$$

$$\mathbf{P}_{XZ} = \sum_{i=1}^{2n+1} \left[\omega_c^i (\mathbf{x}_{k/(k-1),i} - \hat{\mathbf{x}}_{k/(k-1)}) \times (\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)})^T \right] \quad (57)$$

$$\mathbf{K}_k = \mathbf{P}_{XZ} \mathbf{P}_{ZZ}^{-1} \quad (58)$$

2.2 非加性噪声情况

对系统式(21)和式(22)进行 SUKF 滤波,滤波初始化条件由式(23)~式(27)计算。

1) 时间更新

Sigma 点集 $\{\mathbf{x}_i^a\}$, 当 $k=1,2$ 时

$$\mathbf{x}_{k-1,i}^a =$$

$$\begin{cases} \hat{\mathbf{x}}_{k-1}^a - [\sqrt{(n+\lambda)\mathbf{P}_{k-1}^a}]_i & i = 1, 2, \dots, n \\ \hat{\mathbf{x}}_{k-1}^a & i = n+1 \\ \hat{\mathbf{x}}_{k-1}^a + [\sqrt{(n+\lambda)\mathbf{P}_{k-1}^a}]_i & i = n+2, \dots, 2n+1 \end{cases} \quad (59)$$

$$\mathbf{x}_{k/(k-1),i} = f(\mathbf{x}_{k-1,i}^x, \mathbf{x}_{k-1,i}^w) \quad (60)$$

当 $k \geq 3$ 时

$$\mathbf{x}_{k-2,i}^a =$$

$$\begin{cases} \hat{\mathbf{x}}_{k-2}^a - [\sqrt{(n+\lambda)\mathbf{P}_{k-2}^a}]_i & i = 1, 2, \dots, n \\ \hat{\mathbf{x}}_{k-2}^a & i = n+1 \\ \hat{\mathbf{x}}_{k-2}^a + [\sqrt{(n+\lambda)\mathbf{P}_{k-2}^a}]_i & i = n+2, \dots, 2n+1 \end{cases} \quad (61)$$

$$\mathbf{x}_{k-1,i}^a =$$

$$\begin{cases} \hat{\mathbf{x}}_{k-1}^a - [\sqrt{(n+\lambda)\mathbf{P}_{k-1}^a}]_i & i = 1, 2, \dots, n \\ \hat{\mathbf{x}}_{k-1}^a & i = n+1 \\ \hat{\mathbf{x}}_{k-1}^a + [\sqrt{(n+\lambda)\mathbf{P}_{k-1}^a}]_i & i = n+2, \dots, 2n+1 \end{cases} \quad (62)$$

$$\mathbf{x}_{k/(k-1),i} = f(\mathbf{x}_{k-1,i}^x, \mathbf{x}_{k-1,i}^w) + \mathbf{x}_{k-1,i}^x - f(\mathbf{x}_{k-2,i}^x, \mathbf{x}_{k-2,i}^w) \quad (63)$$

式中 n 为扩维后状态向量 $\mathbf{x}_{k-1}^a$ 的维度。

状态一步预测值

$$\hat{\mathbf{x}}_{k/(k-1)} = \sum_{i=1}^{2n+1} (\omega_m^i \mathbf{x}_{k/(k-1),i}) \quad (64)$$

一步预测误差方差矩阵

$$\mathbf{P}_{k/(k-1)} = \sum_{i=1}^{2n+1} \left[\omega_c^i (\mathbf{x}_{k/(k-1),i} - \hat{\mathbf{x}}_{k/(k-1)}) \times (\mathbf{x}_{k/(k-1),i} - \hat{\mathbf{x}}_{k/(k-1)})^T \right] \quad (65)$$

2) 量测更新

状态估计值

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k/(k-1)} + \mathbf{K}_k (\mathbf{Z}_k - \hat{\mathbf{Z}}_{k/(k-1)}) \quad (66)$$

估计误差方差矩阵

$$\mathbf{P}_k = \mathbf{P}_{k/(k-1)} - \mathbf{K}_k \mathbf{P}_{ZZ} \mathbf{K}_k^T \quad (67)$$

式中

$$\mathbf{Z}_{k/(k-1),i} = \mathbf{h}(\mathbf{x}_{k/(k-1),i}, \mathbf{x}_{k,i}^v) \quad (68)$$

$$\hat{\mathbf{Z}}_{k/(k-1)} = \sum_{i=1}^{2n+1} (\omega_m^i \mathbf{Z}_{k/(k-1),i}) \quad (69)$$

$$\mathbf{P}_{ZZ} = \sum_{i=1}^{2n+1} \left[\omega_c^i (\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)}) \times (\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)})^T \right] \quad (70)$$

$$\mathbf{P}_{XZ} = \sum_{i=1}^{2n+1} \left[\omega_c^i (\mathbf{x}_{k/(k-1),i} - \hat{\mathbf{x}}_{k/(k-1)}) \times (\mathbf{Z}_{k/(k-1),i} - \hat{\mathbf{Z}}_{k/(k-1)})^T \right] \quad (71)$$

$$\mathbf{K}_k = \mathbf{P}_{XZ} \mathbf{P}_{ZZ}^{-1} \quad (72)$$

3 算例仿真验证

为了验证本文方法的有效性,采用强非线性系统故障诊断研究中典型的单变量不稳定增长模型^[21-22],其具体形式如下所示:

$$\begin{cases} X_k = 0.5X_{k-1} + 1.5 \frac{X_{k-1}}{1 + X_{k-1}^2} + 3\cos k + b_{k-1} + w_{k-1} \\ Y_k = 0.05X_k^2 + v_k \end{cases}$$

式中 X_k, Y_k 分别表示系统状态值和量测值, w_k, v_k 分别是满足高斯分布的零均值系统误差与量测误差,且分别对应方差 $Q_k=0.1, R_k=0.1, b_k$ 为状态方程中的未知输入项,其取值为

$$b_k = \begin{cases} 0 & 1 \leq k \leq 50 \\ 10 & 51 \leq k \leq 100 \end{cases}$$

分别采用无迹 Kalman 方法与本文 SRF 与 SUKF 方法进行滤波,取滤波初始值 $X_0=3$,误差方差初始值 $P_0=10$,对 UKF 与 SUKF 方法取滤波参数 $\alpha=0.6, \beta=2$ 。3 种方法的滤波效果如图 1 所示。

为进一步验证方法的有效性,对系统进行 500 次蒙特卡洛仿真模拟,分别求得 SRF、SUKE 与 UKF 方法的方均根误差(root mean square er-

ror, RMSE)如图 2 所示。

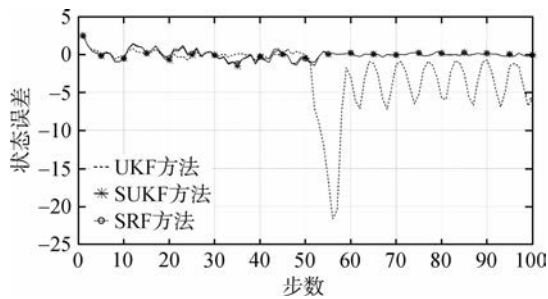


图 1 UKF、SUKF 与 SRF 方法状态误差比较
Fig. 1 State error comparison of UKF, SUKF and SRF methods

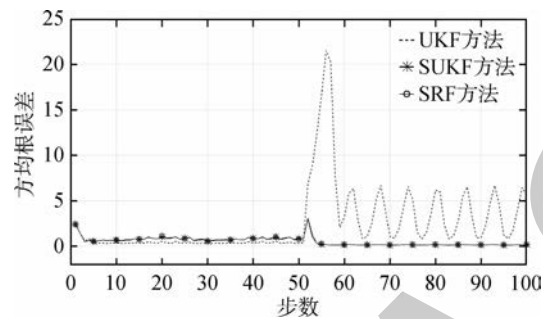


图 2 UKF、SUKF 与 SRF 方法方均根误差比较
Fig. 2 RMSE comparison of UKF, SUKF and SRF methods

3 种方法对应的方均根误差的均值如表 1 所示。

表 1 UKF、SUKF 与 SRF 方法方均根误差均值比较
Table 1 RMSE mean comparison of UKF, SUKF and SRF methods

方法	方均根误差均
SRF	0.532 1
SUKF	0.536 6
UKF	2.591 8

由图 1、图 2 与表 1 的计算结果可以看出,通过对未知输入进行补偿,本文非线性状态方程自校准滤波方法(SRF 方法和 SUKF 方法)可以比传统 UKF 方法更好地跟踪系统状态值的变化,滤波精度由 2.59 提升了近 80%,且估计结果稳定,系统应对未知输入干扰的能力显著提升,具有鲁棒性。

4 结 论

1) 工程实际中,由于环境因素的影响、模型

和参数选取不当等等因素的影响,状态方程常常含有未知输入,导致滤波误差较大。本文方法可以有效地对状态方程中的未知输入进行估计和补偿,提高滤波精度。

2) 自校准无迹 Kalman 滤波(SUKF)主要适用于高斯分布的非线性系统滤波。

3) 自校准秩滤波(SRF)对任意分布情况非线性系统滤波均适用。

4) SRF 与 SUKF 都具有很好的鲁棒性,大多数情况 SRF 比 SUKF 有更好的滤波效果。

5) 本文滤波递推算法,计算简单,便于工程应用。

参考文献:

[1] KALMAN R E. A new approach linear filtering and prediction problems[J]. Journal of Fluids Engineering, 1960, 82(1): 35-45.

[2] SUNAHARA Y. An approximate method of state estimation for nonlinear dynamical systems[J]. Journal of Basic Engineering, 1970, 92(2): 385-393.

[3] BELLANTONI J, DODGE K. A square root formulation of the Kalman-Schmidt filter[J]. AIAA Journal, 1967, 5(7): 1309-1314.

[4] BUCY R S, SENNE K D. Digital synthesis of non-linear filters[J]. Automatica, 1971, 7(3): 287-298.

[5] FUJIMOTO O, OKITA Y, OZAKI S. Nonlinearity compensation extended Kalman filter for target motion analysis[R]. Tokyo: the Institute of Electronics, Information and Communication Engineers (IEICE) General Conference, 1996.

[6] WAN E A, MERWE R V D, NELSON A T. Dual estimation and the unscented transformation[R]. Denver: the 13th Annual Conference on Neural Information Processing Systems, 2000.

[7] JULIER S J, UHLMANN J K, DURRANT-WHYTE H F. A new method for the nonlinear transformation of means and covariances in filters and estimators[J]. IEEE Transactions on Automatic Control, 2000, 45(3): 477-482.

[8] JULIER S J, UHLMANN J K. Unscented filtering and nonlinear wstimation[J]. Proceedings of the IEEE, 2004, 92(3): 401-423.

[9] SIMON D. Optimal state estimation. kalman H_{∞} and non-linear approaches [M]. Hoboken, US: John Wiley and Sons, 2006.

[10] PITT M, SHEPHARD N. Filtering via simulation: auxiliary particle filters[J]. Journal of the American Statistical Association, 1997, 94(446): 590-599.

[11] DOUCET A, GORDON N J, KRISHNAMURTHY V. Particle filters for state estimation of jump Markov linear systems[J]. IEEE Transactions on Signal Processing, 2002, 49(3): 613-624.

[12] CARPENTER J, CLIFFORD P, FEARNHEAD P. Improved particle filter for nonlinear problems[J]. IEE Proceedings, Radar, Sonar and Navigation, 1999, 146(1): 2-7.

[13] 傅惠民, 肖强, 吴云章, 等. 秩滤波方法[J]. 机械强度, 2014, 36(4): 521-526.

FU Huimin, XIAO Qiang, WU Yunzhang, et al. Rank filter method[J]. Journal of Mechanical Strength, 2014, 36(4): 521-526. (in Chinese)

[14] 傅惠民. 连续分布百分位值和百分率的区间估计和假设检验[J]. 航空学报, 1993, 14(11): 569-577.

FU Huimin. Interval estimation and hypothesis test for percentiles and percentages of continuous distributions[J]. Acta Aeronautica et Astronautica Sinica, 1993, 14(11): 569-577. (in Chinese)

[15] 傅惠民. 不完全数据秩分布理论[J]. 航空学报, 1993, 14(11): 578-584.

FU Huimin. Theory of incomplete data rank distributions [J]. Acta Aeronautica et Astronautica Sinica, 1993, 14(11): 578-584. (in Chinese)

[16] 傅惠民. 二维单侧容忍系数方法[J]. 航空学报, 1993, 14(3): 167-172.

FU Huimin. A method of two-dimensional one-sided tolerance factors[J]. Acta Aeronautica et Astronautica Sinica, 1993, 14(3): 167-172. (in Chinese)

[17] BLANLE M, KINNAERT M, LUNZE J, et al. Diagnosis and fault-tolerant control[M]. Berlin, Germany: Springer, 2006.

[18] CHEN J, PATTON R. Robust model-based fault diagnosis for dynamic systems[M]. Norwell, MA, US: Kluwer Academic Publishers, 1999.

[19] GILLIJNS S, MOOR B D. Unbiased minimum variance input and state estimation for linear discrete-time systems [J]. Automatic, 2007, 43(1): 111-116.

[20] 傅惠民, 吴云章, 娄泰山, 等. 自校准 Kalman 滤波方法[J]. 航空动力学报, 2014, 29(6): 1363-1368.

FU Huimin, WU Yunzhang, LOU Taishan, et al. self-calibration Kalman filter method [J]. Journal of Aerospace Power, 2014, 29(6): 1363-1368. (in Chinese)

[21] 李继刚, 陈华秋, 徐春红, 等. 浅述多元数据处理的一种简化拟合形式及算法剖析[J]. 海洋技术, 2002, 21(3): 1-4.

LI Jigang, CHEN Huaqiu, XU Chunhong, et al. A simplified fitting algorithm in multi-variable data processing[J]. Ocean Technology, 2002, 21(3): 1-4. (in Chinese)

[22] 董鑫, 欧阳高翔, 韩威华, 等. 强跟踪 CKF 算法及其在非线系统故障诊断中的应用[J]. 信息与控制, 2014, 43(4): 451-456.

DONG Xin, OUYANG Gaoxiang, HAN Weihua, et al. Strong tracking CKF and its application to fault diagnosis of nonlinear systems[J]. Information and Control, 2014, 43(4): 451-456. (in Chinese)

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